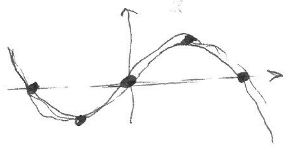
NumMód 2.13 1.24 2011/2012 tavasz 2012.03.20. László Lajos-féle

1. Lagrange-interpolácció $x=(-2,-1,0,1,2)$ $y=(0,-1,0,1,0)$ szorzat alakban

-2	$\boxed{0}$	$\boxed{-1}$	$\boxed{1}$	$\boxed{-\frac{1}{3}}$	$\boxed{0}$
-1	-1	1	0	$-\frac{1}{3}$	
0	0	1	-1		
1	1	-1			
2	0				

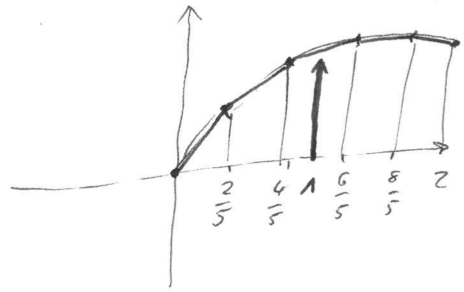


$$p(x)=0-1(x+2)+1(x+2)(x+1)-\frac{1}{3}x(x+1)(x+2)+0\ldots=(x+2)\left[-1+(x+1)-\frac{1}{3}x(x+1)\right]=$$

$$=(x+2)\left[-\frac{1}{3}x^2+\frac{2}{3}x\right]=-\frac{1}{3}x(x+2)(x-2)$$

2. sin $[0,2]$-n $h=\frac{2}{5}$ lépésközzel interpolálva $|\text{hiba}|\le 2\cdot10^{-5}$ az 1 pontban

$$|p(x)-\sin x|\le\frac{M_6}{6!}|\omega_6(x)|\Big|_{x=1}\qquad 6!=720$$



$$(\sin(x))^{(6)}=\ldots\le 1$$

$$\omega_6(x)=(x-0)(x-\tfrac{2}{5})(x-\tfrac{4}{5})(x-\tfrac{6}{5})(x-\tfrac{8}{5})(x-2)$$

$$|\omega_6(1)|=1\cdot\frac{3}{5}\cdot\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{3}{5}\cdot1=\frac{9}{625}$$

$$\frac{1}{720}\cdot\frac{9}{625}=\frac{1}{80\cdot625}=\frac{1}{50000}=0{,}00002$$

3. $f(x)=x^6$ Hermite $x_i:\{0,1,2\}$ $m=\{3,2,1\}$ $f'(x)=6\cdot x^5$ $f''(x)=30x^4$ x^2-2x+1

0	$\boxed{0}$	$\boxed{0}$	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{4}$
0	0	0	1	4	11	
0	0	1	5	26		
1	1	6	57			
1	1	63				
2	64					

$$p(x)=0+0\cdot x+0\cdot x^2+1\cdot x^3+3\cdot x^3(x-1)+4x^3(x-1)^2=$$

$$=x^3+3x^4-3x^3+4x^5-8x^4+4x^3=$$

$$=\underline{4x^5-5x^4+2x^3}$$

Hibatétellel

$$p(x)-f(x)=\frac{M_6(\xi)}{6!}\omega(x)\qquad f^{(6)}=6!\qquad p(x)=f(x)-\omega(x)$$

$$\omega(x)=x^3(x-1)^2(x+2)=x^3(x^2-2x+1)(x+2)=x^3(x^3-4x^2+5x-2)=x^6-4x^5+5x^4-2x^3$$

4. Spline, másodfokú $x=\{-1,0,1,2\}$, $y=\{1,0,1,1\}$, $s'(-1)=-3$

$[-1,0]$:

-1	1	-3	2	
-1	1	-1		
0	0			

$$p_1(x)=1-3(x+1)+2(x+1)^2=$$
$$=1-3x-3+2x^2+4x+2=$$
$$=\underline{2x^2+x}\qquad p_1'(x)=4x+1\qquad p_1'(0)=1$$

$[0,1]$:

0	0	1	0	
0	0	1		
1	1			

$$p_2(x)=0+1\cdot x+0\cdot x^2=\underline{x}\qquad p_2'(x)=1\qquad p_2'(1)=1$$

$[1,2]$:

1	1	1	-1	
1	1	0		
2	1			

$$p_3(x)=1+1(x-1)-1(x-1)^2=1+x-1-x^2+2x-1=\underline{-x^2+3x-1}$$

5. $c = \min_{a,b,c,d} \max_{x\in[-1,1]} \left|\underbrace{x^4+ax^3+bx^2+cx+d}_{t_4(x)}\right|$

$T_0(x)=1,\quad T_1(x)=x,\quad T_2(x)=2x^2-1,\quad T_3(x)=4x^3-3x,\quad T_4(x)=8x^4-8x^2+1$

$a=0,\ b=-1,\ c=0,\ d=\frac{1}{8},\ e=\frac{1}{8}$ $\qquad t_4(x)=x^4-x^2+\frac{1}{8}$ $\qquad \|t_4\|_\infty=\frac{1}{8}$

6. Cseb-pol másodrendű homogén lin. diff.-egyenlete:

$(1-x^2)T_n''-xT_n'+n^2T_n=0$

$n=4$ $\qquad (1-x^2)(8x^4-8x^2+1)''-x(8x^4-8x^2+1)'+16(8x^4-8x^2+1)=$

$=(1-x^2)(32x^3-16x)'-x(32x^3-16x)+128x^4-128x^2+16=$

$=(1-x^2)(96x^2-16)\ \underline{-32x^4+16x^2+128x^4-128x^2+16}=$

$=96x^2-16-96x^4+16x^2+\underline{96x^4-112x^2+16}=0$

def. $\quad T_n=\cos(n\arccos x)$ $\qquad (\cos x)'=-\sin x$ $\qquad (\arccos x)'=\frac{-1}{\sqrt{1-x^2}}$ $\qquad x\in[-1,1]$

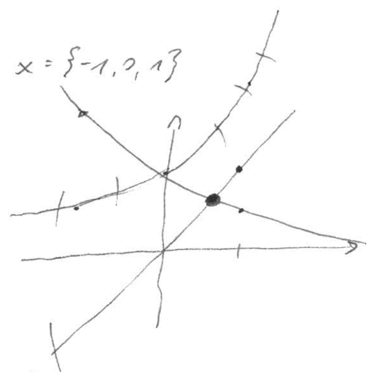
$T_n'=+\sin(n\arccos x)\cdot\frac{+n}{\sqrt{1-x^2}}$

$T_n''=\cos(n\arccos x)\cdot\frac{-n}{\sqrt{1-x^2}}\cdot\frac{n}{\sqrt{1-x^2}}+\sin(n\arccos x)\cdot\frac{+n\cdot 2x}{2\sqrt{(1-x^2)^3}}$

$(1-x^2)T_n''-xT_n'+n^2T_n=-n^2\cos(n\arccos x)-\sin(n\arccos x)\cdot\frac{nx}{2\sqrt{1-x^2}}-$

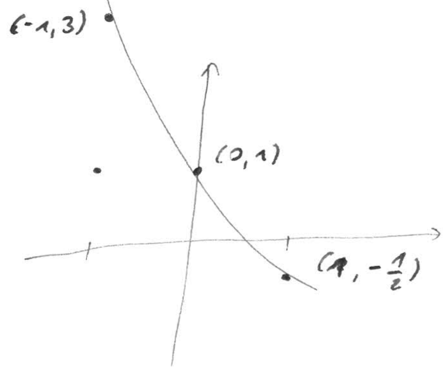
$-\sin(n\arccos x)\cdot\frac{nx}{\sqrt{1-x^2}}+n^2\cos(n\arccos x)=0$ ✓

7. Inverz int. $\quad x=2^{-x}$ $\quad (f(x)=2^{-x}-x=0)$ $\quad [0,1]$-beli gyök $\quad x=\{-1,0,1\}$



$p(y):\quad p(-\frac{1}{2})=1,\ p(1)=0,\ p(3)=-1$

$$\begin{array}{cccc}
-\frac{1}{2} & 1 & & \\
 & & \frac{0-1}{1-(-\frac{1}{2})}=-\frac{2}{3} & \\
 & & & \frac{-\frac{1}{2}+\frac{2}{3}}{3+\frac{1}{2}}=\frac{\frac{1}{6}}{\frac{7}{2}}=\frac{1}{21} \\
1 & 0 & & \\
 & & \frac{-1-0}{3-1}=-\frac{1}{2} & \\
3 & -1 & &
\end{array}$$



$p(y)=1-\frac{2}{3}(y+\frac{1}{2})+\frac{1}{21}(y+\frac{1}{2})(y-1)$

$p(0)=1-\frac{2}{3}\cdot\frac{1}{2}+\frac{1}{21}\cdot\frac{1}{2}\cdot(-1)=1-\frac{1}{3}+\frac{1}{42}=\frac{42-14-1}{42}=$

$=\frac{27}{42}=\frac{9}{14}$