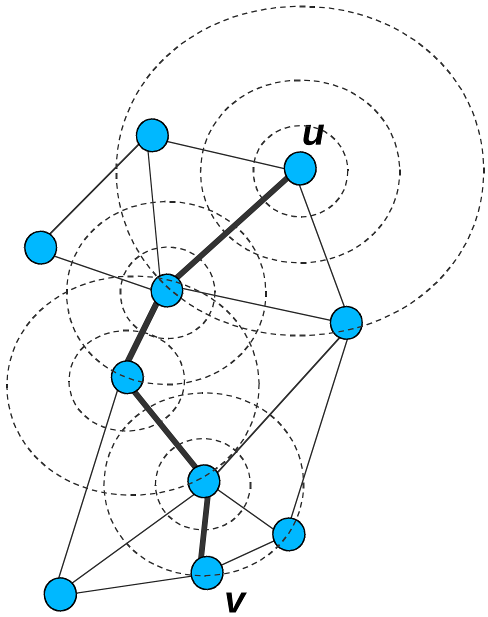
Topology Control

Topology Control

- Sparse topologies, low node degree
  - Storage complexity, storage efficiency
- Short paths, low energy paths
  - Energy: battery life time
    health issues
    (high frequency radiation)
- Low load
- Efficient distributed construction and maintenance
  scalability
  fault tolerance
  self-reconstruction



Graph Properties – Yao Type Topologies

$$\mathrm{SymmY}(V) \subseteq \mathrm{SparsY}(V) \subseteq \mathrm{Yao}(V)$$
$$\mathrm{SparsY}(V) \subseteq \mathrm{BoundY}(V).$$

Topology	Yao	BoundY	SparsY	SymmY
in-degree	n-1	$(k+1)^2$	k	k
out-degree	k	k	k	k
degree	n-1+k	$(k+1)^2+k$	2k	k

Graph Properties

Theorem [Ruppert & Seidel]**:** Let V 2 R^2 be a set of n nodes. Then, for k > 6, Y(V) is a c-spanner, where

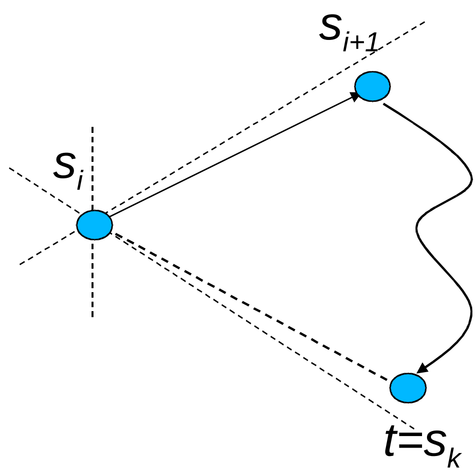
$$c = \frac{1}{1 - 2\sin(\Theta/2)}$$

Proof:

For an arbitrary pair of nodes s,t, let P=($s=s_0, s_1, \ldots, s_k$=t) be the path from s to t, where s_{i+1}, i ¸0, is defined as:

- s_{i+1} = t, if t is neighbor of s_i in Y(G),
- otherwise, s_{i+1} is the neighbor of s_i in the sector around s_i which contains t.

This procedure defines a routing, called **sector routing**



Graph Properties

Theorem [Ruppert & Seidel]**:** Let V 2 R^2 be a set of n nodes. Then, for k > 6, Y(V) is a c-spanner, where

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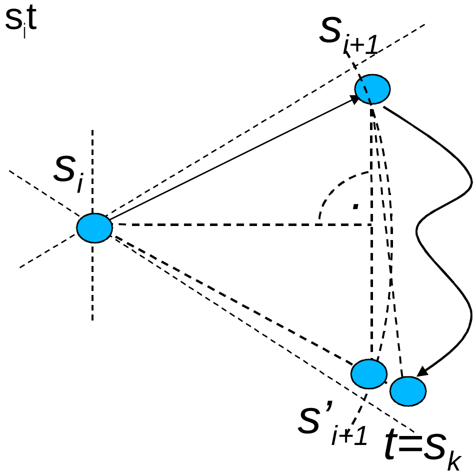
Proof (cont.):

Let s'_{i+1} be the point contained in the line segment $s_i t$ with $||s'_i,s_{i+1}|| = ||s_i,s_{i+1}||$.



(1) $||s_{i+1},t||$ · $||s_{i+1},s'_{i+1}|| + ||s'_{i+1},t||$
$= ||s_{i+1},s'_{i+1}|| + ||s_i,t|| - ||s_i,s'_{i+1}||$.

(2) $||s'_{i+1},s_{i+1}||$ · $2 \sin(\Theta/2)||s_i,s_{i+1}||$.

From (1) and (2) we obtain

(3) $||s_i,t|| - ||s_{i+1},t||$ ¸ $(1 - 2 \sin(\Theta/2))\ ||s_i,s_{i+1}||$.

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Proof (cont.):

(3) $||s_i,t|| - ||s_{i+1},t||$ ¸ $(1 - 2 \sin(\Theta/2))\ ||s_i,s_{i+1}||$.

Summing (3) for i=1,...k-1:

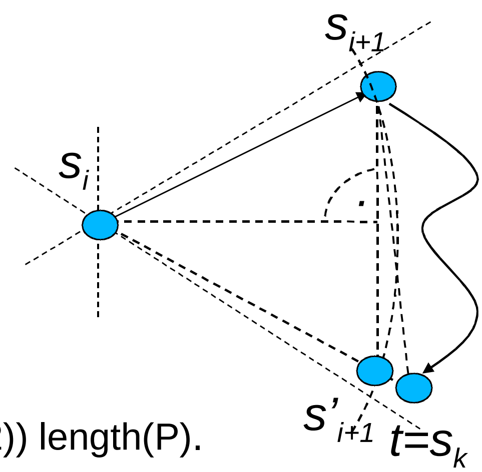
(4) $\sum (||s_i,t|| - ||s_{i+1},t||)$ ¸ $\sum (1 - 2 \sin(\Theta/2))\ ||s_i,s_{i+1}||$.

The left hand side is a telescope sum:

(5) $\sum (||s_i,t|| - ||s_{i+1},t||) = ||s_0,t|| - ||s_k,t|| = ||s,t||$.

We obtain

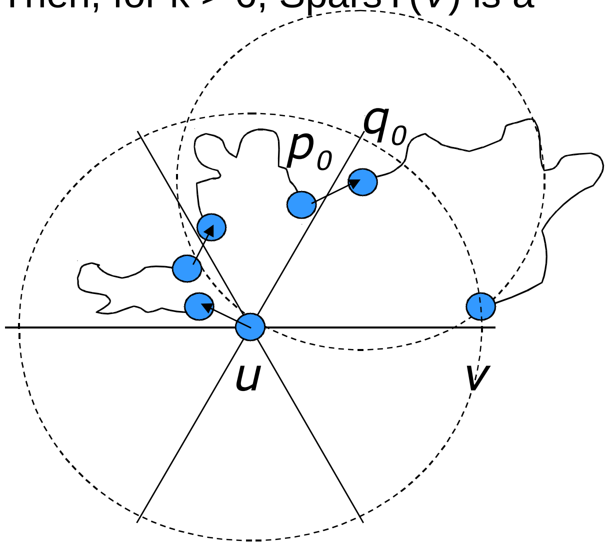
(6) $||s,t||$ ¸ $(1- 2 \sin(\Theta/2)) \sum ||s_i,s_{i+1}|| = (1 - 2 \sin(\Theta/2))$ length(P).



Graph Properties

Theorem*: Let V 2 R^2 be a set of n nodes. Then, for k > 6, SparsY(V) is a weak c-spanner, where

$$c = \frac{1}{1 - 2\ \sin(\Theta/2)}$$



Proof (sketch):
Consider an arbitrary pair of nodes u,v.
Let i be the sector of u containing v.
Let q_0 be the Yao-neighbor of u in i.

- If u has no directed edge in sector i in SparsY(V), then
  - either the sector is empty
  - or the Yao-neighbor q_0 is incident to an edge (w, q_0) 2 E, where w is in another sector of u. Furthermore, $||u, p_0|| < ||u, q_0||$, because $\theta < \pi/3$ and $|| q_0, p_0|| < ||u, q_0 ||$.
- u has at least one neighbor in SparsY(V).

Graph Properties

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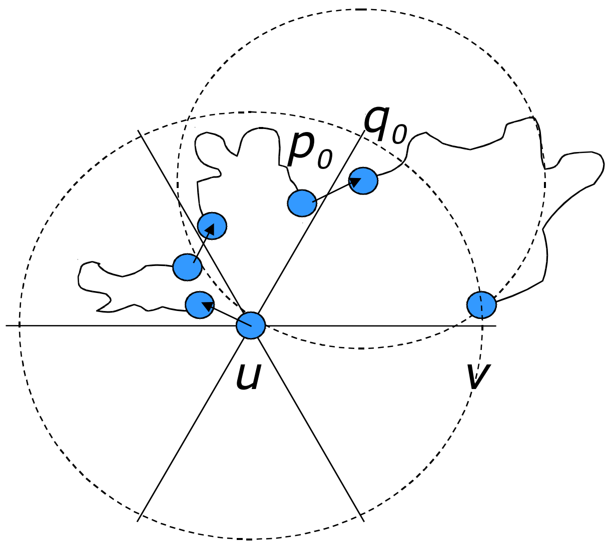

Proof (cont.):

- Recursively construct the path P(u,v):
  - If u=v then P(u,v)=()
  - If (u,v) 2 E then P(u,v) =(u,v)
  - Let i be the sector of u containing v
  - Let q_0 be the Yao-neighbor of u in i
  - If q_0 is not directly connected to u then
    - Recursively construct the path P(u,p_0)
  - Recursively construct P(q_0,v)
  - P(u,v) = P(u, p_0) ○ (p_0,q_0) ○ P(q_0,v)

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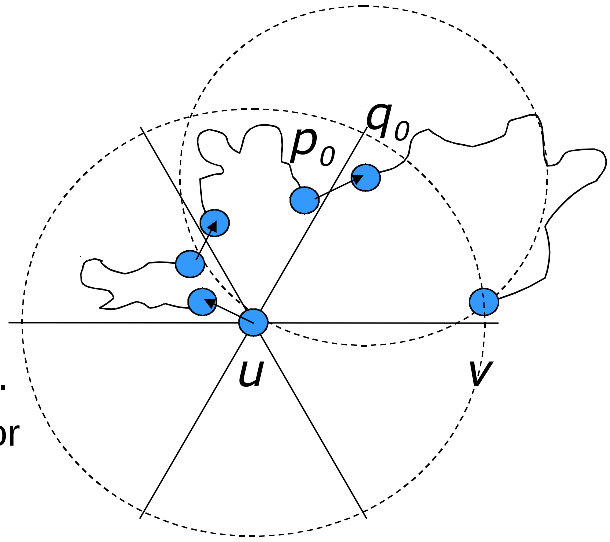


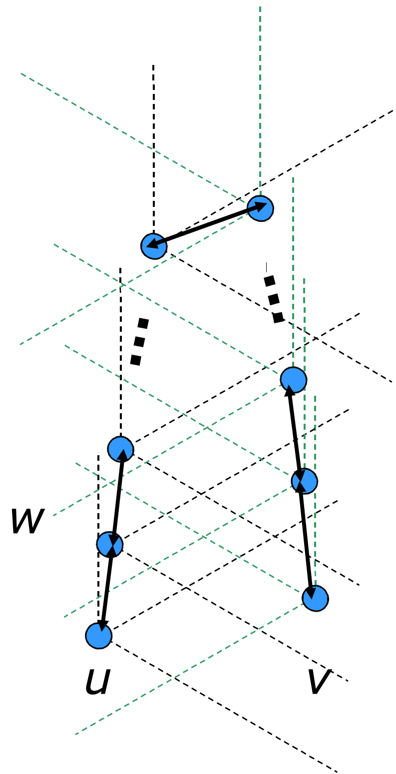
Proof (cont.):

- Recursively construct the path P(u,v).
- The recursion ends when a Yao-neighbor is directly connected to u.
- We have: $||p_0,u|| < ||q_0,u||$ and $||q_0,u||\cdot ||u,v||$.
- Since every node has at least one neighbor in E this process terminates
- All nodes p_i q_i in P(u,q_0) are inside the disk with center u and radius ||u,v||
- It can be shown that the disk amplification containing the whole path P(u,v) can be at most 1 / (1 - 2 sin (θ/2)).

Graph Properties

Theorem*: Let V 2 R^2 be a set of n nodes. Then SymmY(V) is

- connected, if k ≥ 6
- neither a weak c-spanner for any constant c, nor a (c,d)-power spanner



Proof:

- Not a weak/power spanner:
Idea: place the nodes on almost paralell lines.

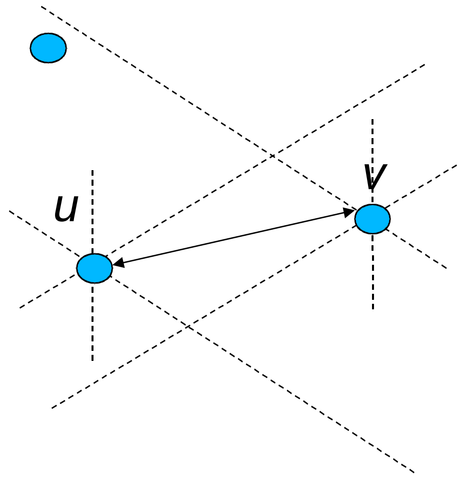
Graph Properties

Theorem*: Let *V* 2 R^2 be a set of *n* nodes. Then SymmY(*V*) is

- connected, if k ≥ 6
- neither a weak *c*-spanner for any constant c, nor a (*c*,*d*)-power spanner

Proof:

- Connectivity: Let (u, v) be a directed edge in the Yao-graph. We show by induction on the edge lengths, that there is a path from u to v in SymmY(V)
- Start of the induction:The closest pair of nodes u,v must be connected by a symmetric edge.



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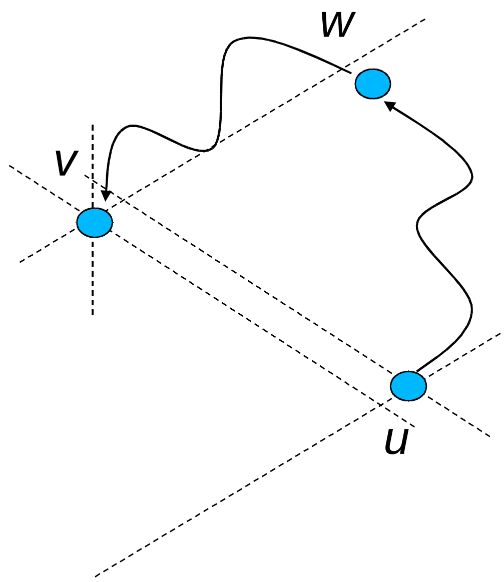
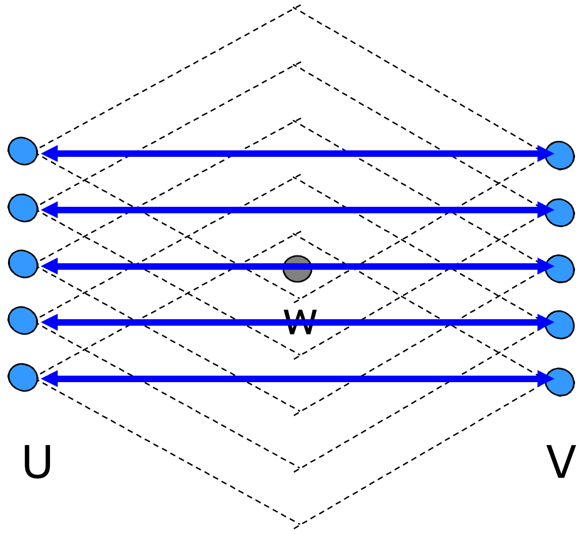
Proof (cont.):

- Connectivity: Induction step:
  Assume, the claim holds for all edges (r,s) of the Yao-graph with ||r,s||<||u,v||.
- Case 1: (v,u) 2 Yao(V) ➔ (u,v) is symmetric.
- Case 2: (v,u) not in Yao(V)
  Then there is a node w in the sector around u containing v with ||u,w|| < ||u,v||.
  By induction, there exists a path from u to w and from w to v in SymmY(V).
  ➔ A path from u to v exists.



Maintaining the Network

Theorem*: Let V be a vertex set. Then $\Theta(|V|)$ edges need to be changed, if an **enter** or **leave** operation takes place in Yao(V), BoundY(V), SparsY(V) or SymmY(V).



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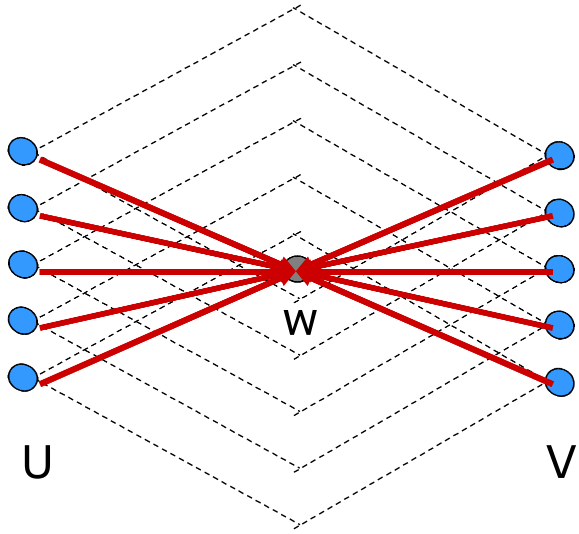

Maintaining the Network

Theorem*: Let *V* be a node set and *m* the number of involved edges. Then the network structure of Yao(*V*), BoundY(*V*), SparsY(*V*) and SymmY(*V*) can be rebuilt in $O(m \log s)$ time.
For Yao(*V*):

enter	**leave**
• Inform the nodes that *u* has entered • Search for next neighbor of *u* in each sector • The informed nodes check empty sectors, whether *u* can be its nearest neighbor • The informed nodes check not-empty sectors, whether *u* is closer	• At some time the node *v* notices that *u* has left the network • *v* informs other nodes that *u* has left • All nodes adjacent to *u* have to determine new neighbors (this is done by a reduced version of the enter-algorithm)

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