Fast Collisionless Pattern Formation by Anonymous, Position-Aware Robots

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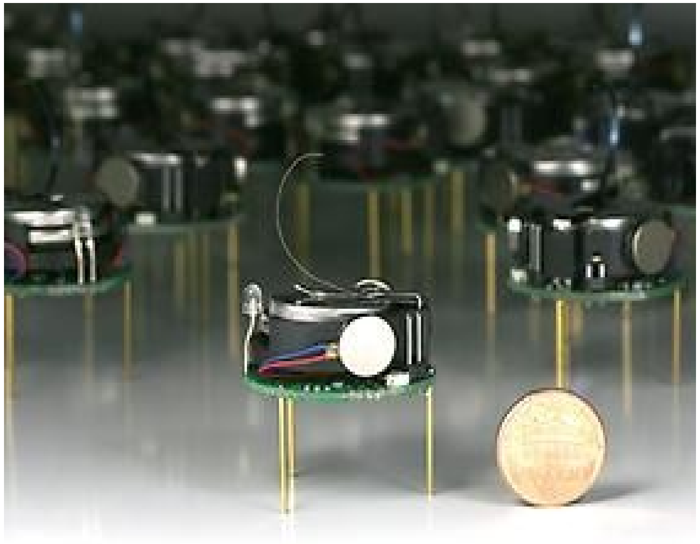
Pattern Formation

- n robots with restricted capabilities
  - Limited viewing range
  - No communication
- 2D plane setting (2D grid)
- They want to form a given connected pattern
  - Special case:
    gather at a given point
    (Point formation)



http://www.eecs.harvard.edu/ssr/projects/progSA/kilobot.html

Where is it needed?

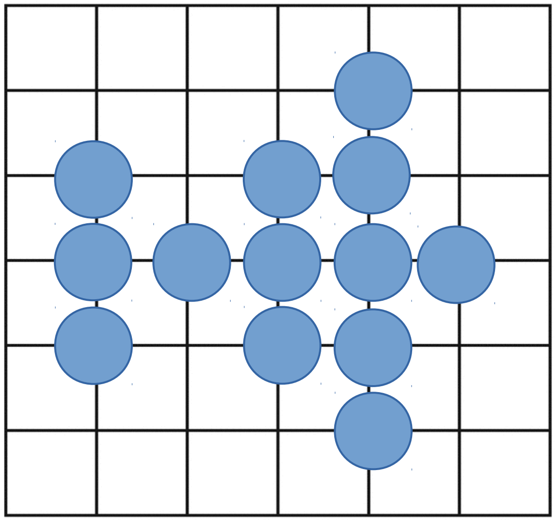
- Gathering:
  - Exploration Robots
    - To exchange data they need to gather
  - Task splitting
    - After gathering the main robot distributes the tasks
  - ...
- Pattern formation
  - Flight array
  - Self-deploying mobile sensors
  - Self-organizing particle system

Overview

- Model
- Problems
- Related work
- Our contribution
- Summary

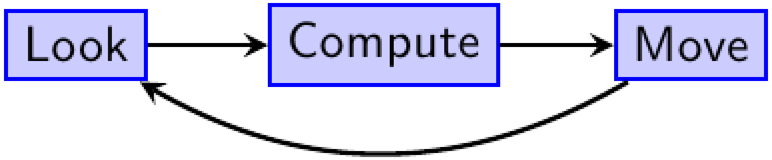
Anonymous Restricted Robots

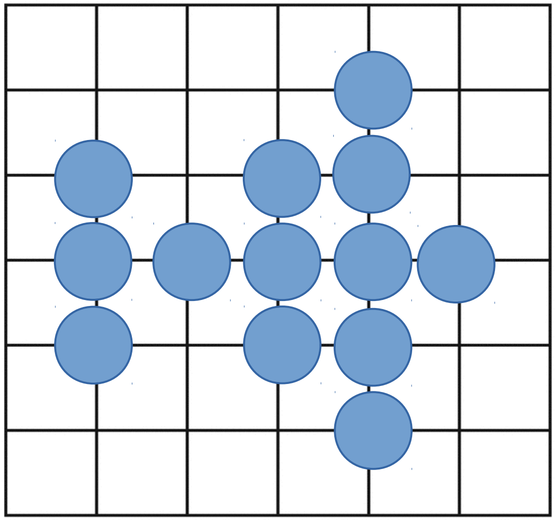
- Identical and anonymous
- Oblivious (for gathering)
  - finite state, i.e. O(1) bits of persistent memory for pattern formation
- No communication
- Limited sight: 2 units
- Represented by disk of unit diameter
- Aware of own position in Z^2
- Common coordinate system
- Can move on the edges of the grid



Synchronous Look-Compute-Move (LCM)

- In each time unit each robot can move to a neighboring grid vertex or stay idle
- Collision is not allowed
- Synchronous Look-Compute-Move cycles

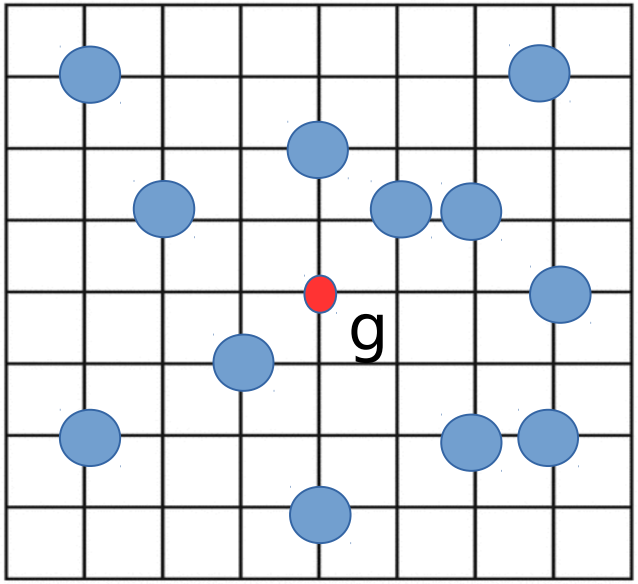




- **Look:** take snapshot of its 2-hop neighborhood
- **Compute:** decision to move or stay in place
- **Move:** moving or staying in place

Problems considered

- **Collisionless Gathering**
  - Given:
    - n robots on different grid vertices
    - gathering vertex g, known for all robots
  - Goal:
    - Form a connected configuration containig g
    - Collision is not allowed



Problems considered

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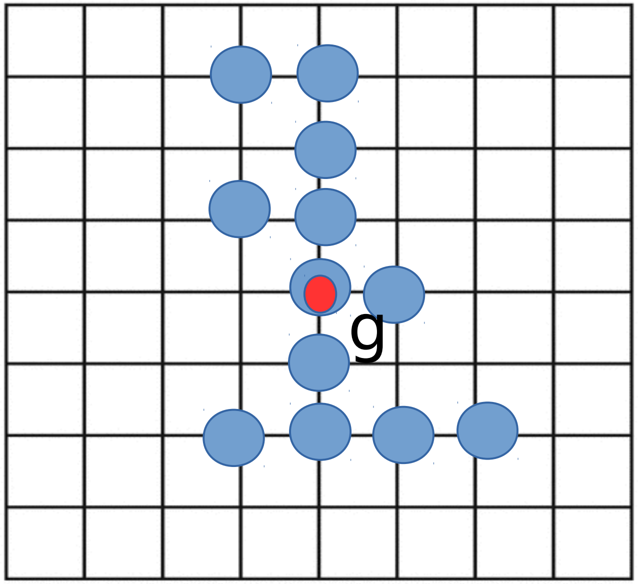
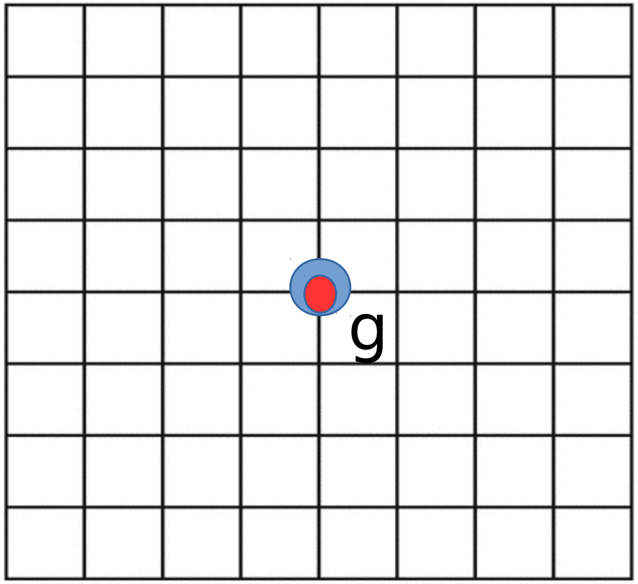


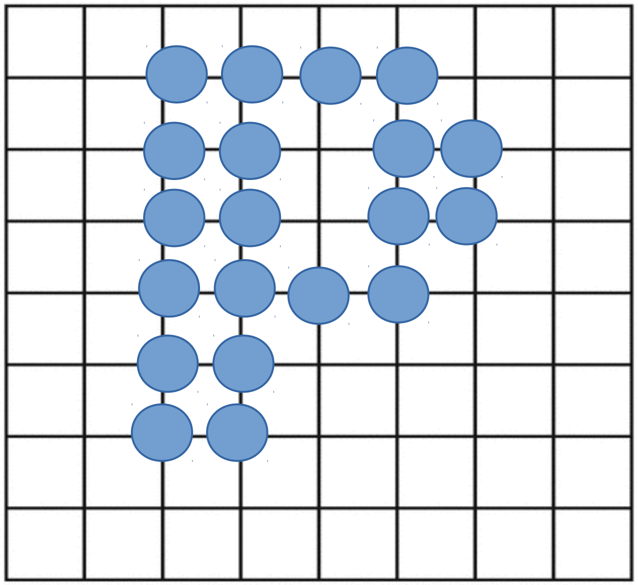
Problems considered

- Helpful intermediate gathering problem:
  Lemmings Problem
  - Given:
    - n robots on different grid vertices
    - vertex g, known for all robots
  - Goal:
    - All robots must occupy g
    - Collision is only allowed at the single vertex g



Problems considered

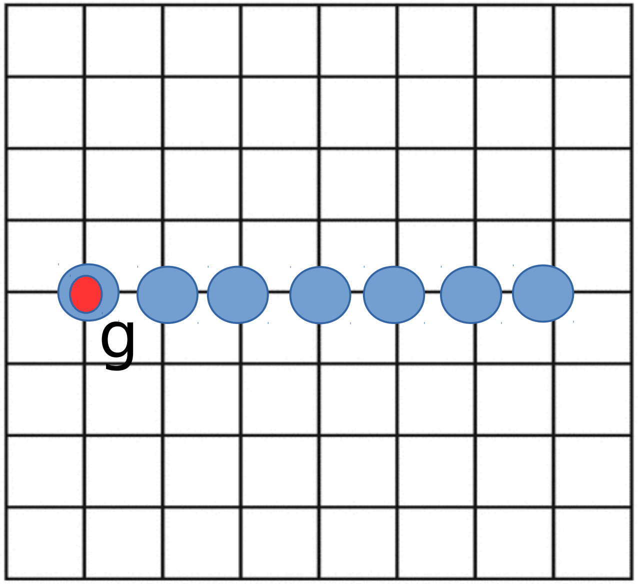
- **Formation of a connected pattern F**
  - Given:
    - n robots on different grid vertices
    - pattern P, known for all robots
      - as set of vertices, or
      - partially described,
        e.g. build a connected pattern
        with minimum diameter
  - Goal:
    - Form P
    - Collision is not allowed



Problems considered

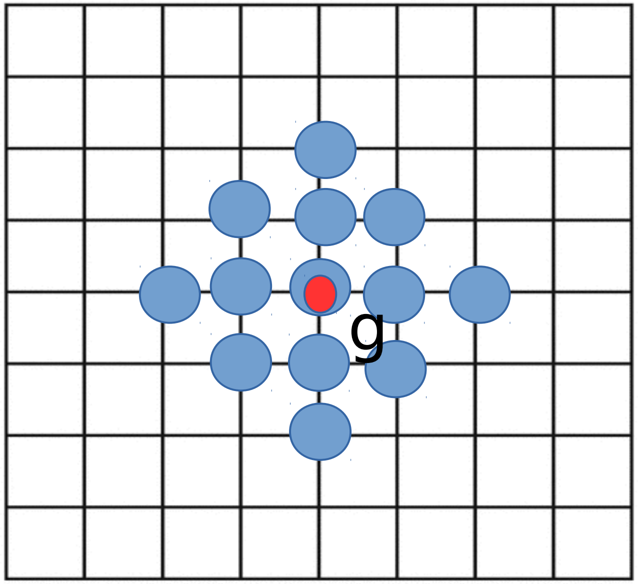
Special case of pattern formation:
Axis parallel line segment formation

- Given:
  - n robots on different grid vertices
  - vertex gknown
    for all robots
- Goal:
  - Form an axis parallel
    line segment with end point g



Problems considered

- Special case of pattern formation: **Focused coverage**
  - Given:
    - n robots on different grid vertices
    - vertex g (Point-of-Interest, POI), known for all robots
  - Goal:
    - Cover all vertices of an area with maximum radius around the POI without holes
    - Collision is not allowed



Related work

- Survey on gathering by oblivious robots [Cileiebak et al. SICOMP 2012]
- Asynchronous gathering of oblivious point-like robots with infinite visibility in LCM model in finite time [Cohen Peleg, SICOMP 2005]
- Gathering of fat robots with infinite visibility in finite time [Czyzowicz et al., TCS 2009]
- Gathering of fat robots on the grid with limited visbility at a given point in time O(nD) time [Chord-Landwehr et al. 2009]

- Formation of geometric patterns by anonymous robots with infinite visibility [Suzuki,Yamashita, SICOMP 1999]
- Pattern formation by robots with limited visbility [Suzuki, Yamashita, SIROCCO 2013]

- Focused coverage by self deploying mobile sensors in finite time [Li et al. TMC 2011]
- Focused coverage in O(nD) time [Blazovics, Lukovszki ALGOSENSORS 2013]

Our Contribution (1)

- **Lemmings Problem:**
  - Algorithm for oblivious robots with visibility range of 2 units solving the problem in

    $2n + D - 1$ time steps,

    - D is the maximum initial distance of a robot from g
  - Lower bound: **$\Omega(n + D)$**
    - Holds also for robots with infinite visibility range
  - Optimal up to a constant factor

Our Contribution (2)

- **Collisionless Gathering:**
  - Algorithm for oblivious robots with visibility range of 2 units solving the problem in
    n + D – 1 time steps
  - Improves previous upper bound of O(nD) [Chord-Landwehr et al. 2011]

Our Contribution (3)

- **Formation of an axis parallel line segment**
  - Algorithm for finite state robots solving the problem in

    3n + D + 3 steps

  - Only 2 bits of persistent memory

Our Contribution (4)

- **Formation of a connected pattern F:**
  - Algorithm for finite state robots solving the problem in
    O(n + D*) time steps,
    where D* is the diameter of the initial scene
    - consisting of F and the initial positions of the robots

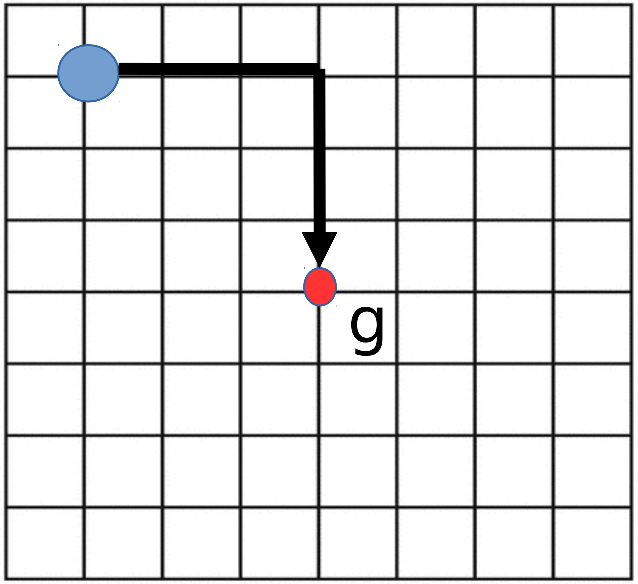
Our Contribution (5)

- **Focused Coverage:**
  - Algorithm for finite state robots solving the problem in
    O(n + D) time steps
  - Improves previous upper bound of O(S), where S is the sum of initial distances of the nodes from g [Blazovics, Lukovszki, 2013]

X-Y-Routing

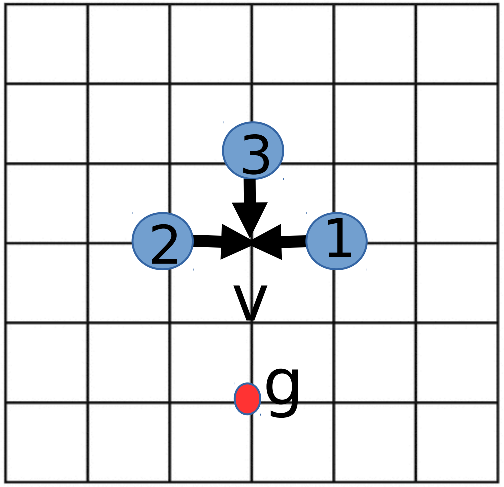
- In each time step each robot wants to decrease its hop distance to g:
  - it moves horizontally until it has the same x-coordinate as g
  - then it moves vertically until it reaches g

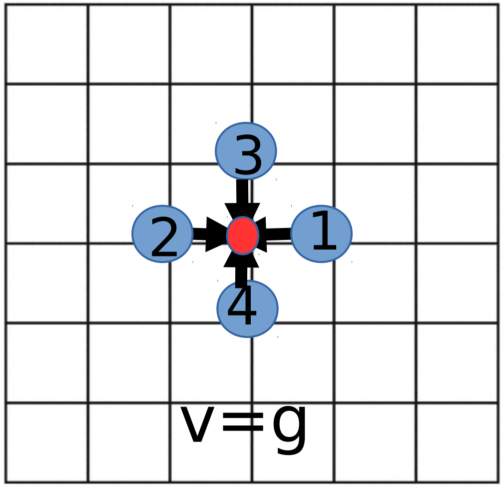
- If the next vertex of the x-y-path is occupied, then stay idle



X-Y-Routing

- If 2 or more robots wants to move to the same vertex v:
  - the robot with highest priority moves to v
  - the other robots stay in place





- Collision free
- Each robot knows its 2-hop neighborhood

==> robots can decide locally, which one can move

- Oblivious

X-Y-Routing

Algorithm 1 x-y-routing(r)

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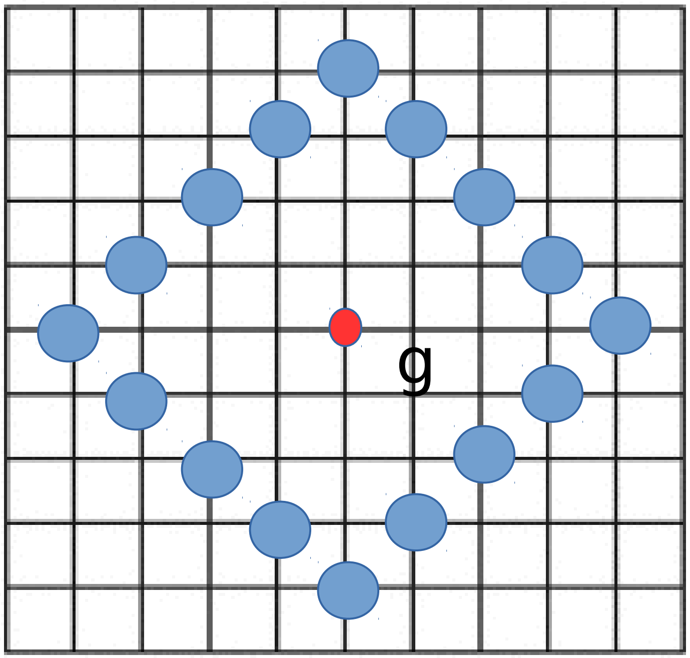
while r has not yet reached g **do**
  $p \leftarrow nexthop(r, t)$
  if p is unoccupied **and** $\nexists$ another robot r' with $nexthop(r', t) = p$, s.t. r' has higher priority than r **then**
    r moves to p
  else
    r stays in place
  end if
  $t \leftarrow t + 1$
end while

Lemmings Problem – Lower Bound

- Each robot must arrive at g
  ==> at least D steps
- In each time step at most 1 robots can reach g
  ==> at least n steps



- **Theorem 1:** Let R be a set of n robots with infinite visibility placed on different vertices of Z^2.
  Each algorithm solving the synchronous lemmings problem needs $\Omega(n + D)$ time steps.

Lemmings Problem – Upper Bound

Perform the x-y-routing algorithm with the modification: treat g as it woud be always an unoccupied one

Theorem 2: Let R be a set of n oblivious robots placed at different vertices of Z^2. Let g be the gathering vertex. By performing the x-y-routing algorithm, all robots reach g in

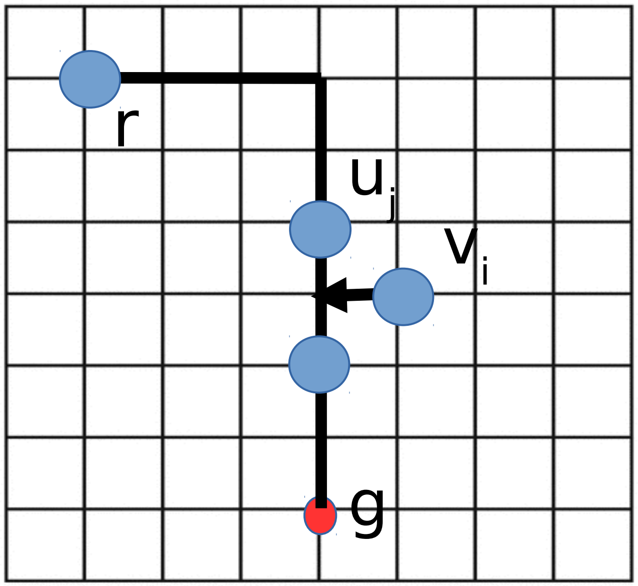
$$2n + D - 1 \text{ time steps,}$$

where D is the maximum initial distance of a robot from g.

Lemmings Problem – Upper Bound

Proof strategy:

- Asume first, all robots are placed on the same x-y-path P terminating at g
- Show by induction: by x-y routing, all robots reach g within n+D-1 time
- Consider the robot r arriving at g as the last one
- Let P be the x-y-path from the initial position of r to g
- Let $U=\{u_1,\ldots,u_k\}$, $u_k=r$ be the robots on P
- If we remove all robots R\U, all robots of U would arrive at g within k+D'-1 steps
- Show that each v_i in R\U can increase the arrival time of r by at most 2 time steps



Collisionless Gathering

Perform the (original) x-y-routing algorithm

Theorem 3: Let R be a set of n oblivious robots placed on different vertices of Z^2. Let g be the gathering vertex. By performing the x-y-routing algorithm, the robots form a connected configuration containing g in

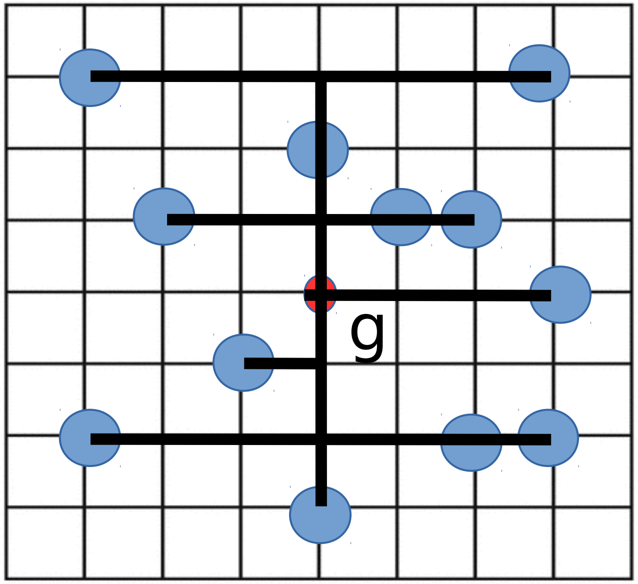
$$n + D - 1 \text{ time steps,}$$

where D is the maximum initial distance of a robot from g.

Collisionless Gathering

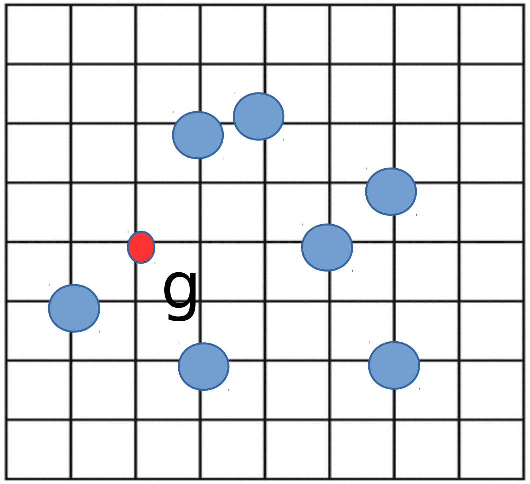
Proof strategy:

- For each robot r, let
  - p_r* be closest point of r to g during the algorithm,
  - P_r* be the x-y path from p_r* to g
- Let T* = $\bigcup_r P_r$*
- Show that
  (i) T* contains g
  (ii) T* has no unoccupied vertex, i. e. T* is connected
  (iii) each vertex of T* becomes occupied in n+D-1 steps
  - By induction, prove that after i+D steps all vertices of T* with hop distance at most i from g are occupied



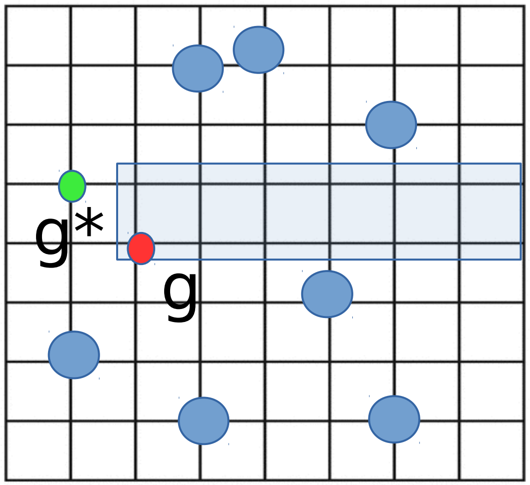
Axis-Parallel Line Segment Formation

1) Each robot with y-coordinate > g_y moves one step upwards
   Each robot with y-coordinate <= g_y moves one step downwards



Axis-Parallel Line Segment Formation

1) Each robot with y-coordinate > g_y moves one step upwards
   Each robot with y-coordinate <= g_y moves one step downwards
2) Let $g^*=(g_x-1,g_y+1)$.
   Execute the Lemmings algorithm with sink g*.
   When a robot r occupies g* in step t, r moves one hop to the right in step t+1

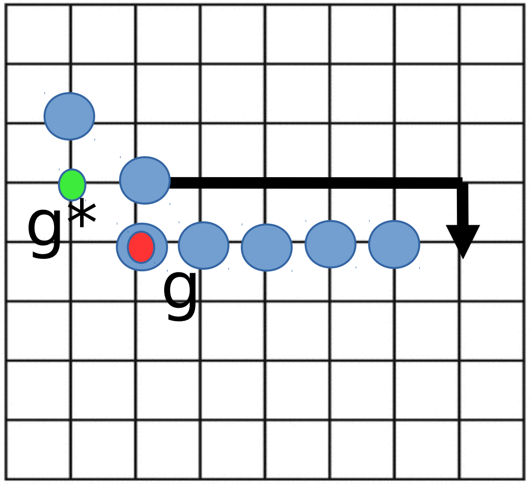


Forming Axis-Parallel Line Segment

1)Each robot with y-coordinate > g_y moves one step upwards
Each robot with y-coordinate <= g_y moves one step downwards

2)Let $g^*=(g_x-1,g_y+1)$.
Execute the Lemmings algorithm with sink g*.
When a robot r occupies g* in step t, r moves one hop to the right in step t+1

3)Build the line segment L as follows:
Until the vertex v bellow the current position of a robot is occupied move to the right. Otherwise, occupy v and terminate the algoritm of that robot



Axis-Parallel Line Segment Formation

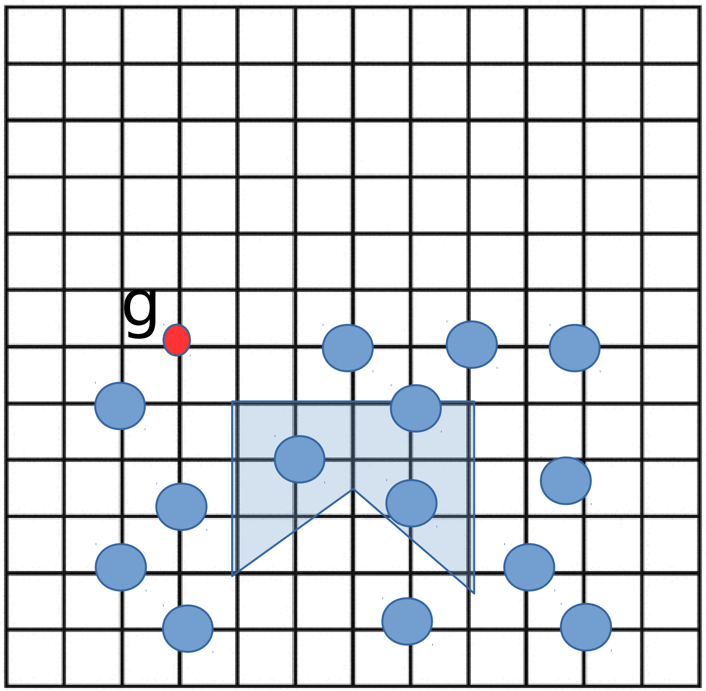
Theorem 4: Let R be a set of n finite state robots placed on different vertices of Z^2. Let g be a point, known for all robots. Then by the above algorithm the robots form a horizontal line segment with left end point g in

$$3n + D + 3 \text{ time steps,}$$

where D is the maximum initial distance of a robot from g.

Forming an Arbitrary Connected Pattern F of size n

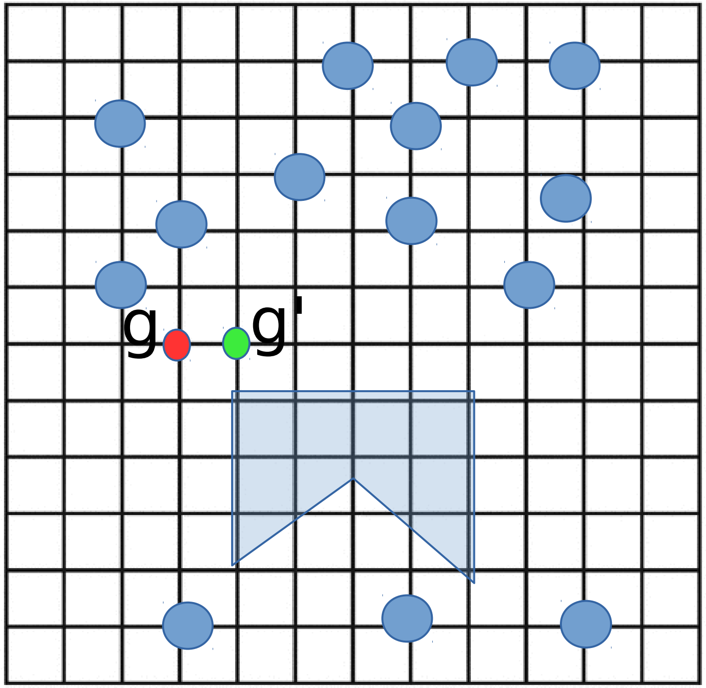
1) Let y_{min} and y_{max} be the minimum and maximum y-coordinate of F. Each robot with y-coordinate >= y_{min} moves y_{max} - y_{min} + 2 steps upwards



Forming an Arbitrary Connected Pattern F of size n

1) Let y_{min} and y_{max} be the minimum and maximum y-coordinate of F. Each robot with y-coordinate >= y_{min} moves $y_{max} - y_{min} + 2$ steps upwards

2) Let $g=(g_x-1, y_{min}-1)$, $g'=(g_x, y_{min}-1)$. Execute the Lemmings algorithm with sink g. When a robot r occupies g in step t, r moves to g' step t+1



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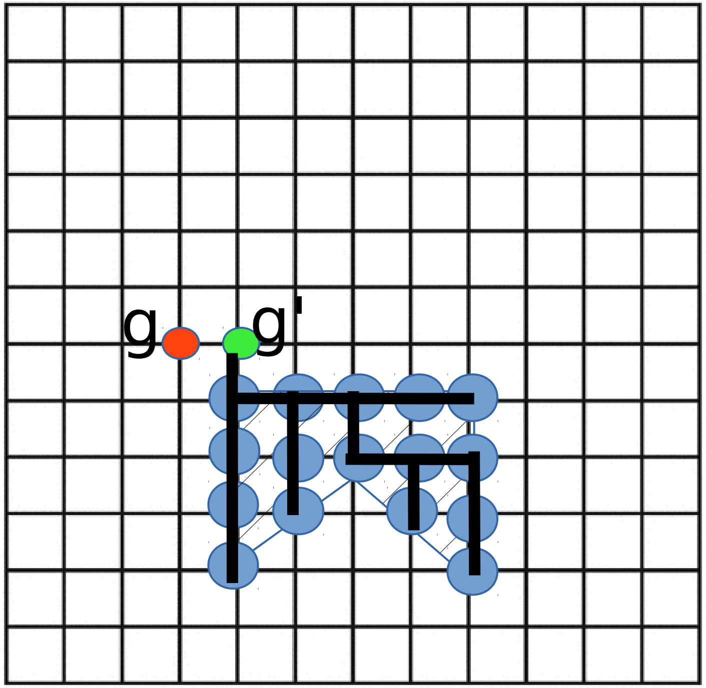
2)Let g=(g_x-1,y_{min}-1), g'=(g_x,y_{min}-1). Execute the Lemmings algorithm with sink g. When a robot r occupies g in step t, r moves to g' step t+1.

3)Let T be a spanning tree of F. Build F from source g' by DFS filling of T, using the arrivals of the robots in g during the Lemmings algorithm as input stream



Forming an Arbitrary Connected Pattern F of size n

Theorem 5: Let R be a set of n finite state robots placed on different vertices of Z^2. Let F be a connected formation, known for all robots. Then the robots form F in time

$$O(n + D^*),$$

where D* is the diameter of the point set containing F and the initial positions of the robots.

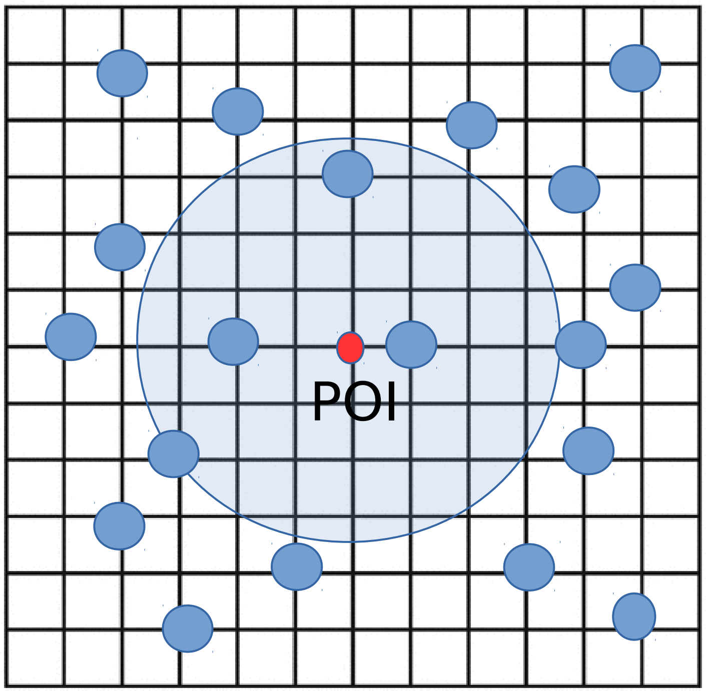
Focused Coverage

- Self deploying mobile sensors
- Solution by oblivious nodes in finite time [Li et al., 2009, 2011]
- O(nD) time [Blazovics, Lukovszki, 2013]

Assume n is known for all nodes

Focused Coverage:

1) All robots compute the disc C centered at the POI with maximum radius, which can be covered by n nodes
2) Perform the pattern formation algorithm for C



Focused Coverage

Corollary: Let R be a set of n finite state mobile nodes placed on different vertices of Z^2. Assume n is known for all nodes. Then the nodes form F in time

$$O(n + D),$$

where D is the maximum initial distance of a node from the POI.

Summary

- Introduced the Lemmings problem in the 2D grid
  - helpful tool for Pattern formation
  - Solved by oblivious robots with visibility range of 2 units in **2n + D – 1** time step
  - Lower bound of **Ω(n + D)** shown
- Collisionless gathering problem by oblivious robots in the 2D grid in **n + D - 1** time steps
  - Improves previous bound of O(nD)
- Axis parallel line segment formation by finite state robots in **3n + D + 3** time steps
- Formation of an arbitrary connected pattern by finite state robots in **O(n + D*)** time steps
- Focused coverage by finite state nodes in **O(n+D)** time
  - Improves previous bound of O(nD)

Thank you!

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