The Accuracy of Power Law based Similarity Model in Phonebook-centric Social Networks

Péter Ekler
Department of Automation and Applied Informatics
Budapest University of Technology and Economics
Magyar Tudósok Körútja 2., 1113 Budapest, Hungary
peter.ekler@aut.bme.hu

Tamás Lukovszki
Faculty of Informatics
Eötvös Loránd University
Pázmány Péter sétány 1/C, 1117 Budapest, Hungary
lukovszki@inf.elte.hu

***Abstract*— Social networks are becoming increasingly popular nowadays. The increasing capabilities of mobile phones enable them to participate in such networks. We should consider the fact, that the phonebooks in the mobile devices represent social relationships that can be integrated in the social networks. Such networks provide a synchronization mechanism between phonebooks of the users and the social network which allows detecting other users listed in the phonebooks. Users can accept detected similarities. After that, if one of their contacts changes her or his personal detail, it will be propagated automatically into the phonebooks, after considering privacy settings. Estimating the total number of these similarities is a key issue from scalability point of view in such networks. We implemented a phonebook-centric social network, called Phonebookmark and investigated the structure of the network. Previously it was shown that the distribution of similarities follows a power law. Also a model was proposed by us, which can be used to calculate the total number of similarities. However the accuracy of the model is another question, because of the infinite variance of the power law distribution. The contribution of this paper is that using the fact that a member of the network can only be involved in a limited number of similarities results in a similarity distribution with a finite variance. Therefore, central limit theorems can be used to show the accuracy of our estimation of the total number of similarities. However the model can be used in other power law distributions which apply to the requirements.**

Keywords-component; social networks, power law distribution, variance, mobile phones

I. INTRODUCTION

In the last decade the internet related technologies developed rapidly. As reasons of this growth new type of solutions and applications have appeared. One of the most popular solutions are the social network sites (SNS). Since their introduction, social network sites such as Facebook, Myspace and LinkedIn have attracted millions of users, many of whom have integrated these sites into their daily practices and they even visit these multiple times per day. These popular online social networks are among the top ten visited websites on the Internet [1]. The basic idea behind such networks is that users can manage personal relationships online on these networks.

According to new statistics [2] Facebook has more than 400 million active users, 50% of the active users log on to Facebook in any given day, more than 35 million users update their status each day and an average user spends more than 55 minutes per day on Facebook. Facebook began in early 2004 and the above statistics show that such popular social networks can have a huge growth which has to be considered during the design of any SNS.

Mobile phones and mobile applications are another hot topic nowadays. Facebook statistics also show that there are more than 65 million active users currently accessing Facebook through their mobile devices. People that use Facebook on their mobile devices are almost 50% more active on Facebook than non-mobile users. The increasing capabilities of mobile devices allow them to participate in social network applications as well. Mobile phone support in general social networks are usually limited mainly to photo and video upload capabilities and access to the social network using the mobile web browser.

However we should consider the fact, that the phonebook of the mobile device also describe the social relationships of its owner. Discovering additional relations in social networks is beneficial for sharing personal data or other content. Given an implementation that allows us to upload as well as download our contacts to and from the social networking application, we can completely keep our contacts synchronized so that we can see all of our contacts on the mobile phone as well as on the web interface. In addition to that if the system detects that some of my private contacts in the phonebook is similar to another registered members of the social network (i.e. may identify the same person), it can discover and suggest social relationships automatically. In the rest of this paper we refer to this solution as a *phonebook-centric social network* (PCSN). Discovering and handling such similarities in phonebook-centric social networks is a key issue. If a member changes some of her or his detail, it should be propagated in every phonebook to which she or he is related after considering privacy settings. In addition to that, with the help of detected similarities the system can keep the phonebooks always up-to-date.

Power law distribution is quite common in social networks and similar internet related graphs as measurements and examples show in Section 2. The number of similarities in phonebook-centric social networks is very important from performance and scalability point of view.

We show that the distribution of similarities can be modeled with a random probability value X with $\Pr[X \geq x] \sim cx^{-\alpha}$, if $x \leq n$ and $\Pr[X \geq x] = 0$ otherwise, where $\alpha>1$.

As a main contribution of this paper, we show that the distribution of similarities has a finite variance which allows us to use the central limit theorem to prove the accuracy of our estimation of the total number of similarities. This model can be used generally in other similar distributions.

As a practical result, the concept of phonebook-centric social networks was applied in the *Phonebookmark* project at Nokia Siemens Networks. *Phonebookmark* is a phonebook-centric social network implementation by Nokia Siemens Networks. We took part in the implementation and before public introduction it was available for a group of general users from April to December of 2008. It had 420 registered members with more than 72000 private contacts, which is a suitable number for analyzing the behavior of the network. During this period we have collected and measured different type of data related to the social network.

The rest of the paper is structured as follows. Section 2 describes related work in the field of social networks and power law distributions. Section 3 introduces the structure of phonebook-centric social networks. Section 4 summarizes our previously published model related to calculating the total number of similarities in the network. Section 5 states a general theorem related to the variance of power law distribution with relevant upper bound and uses it to prove the accuracy of the model described in Section 4. Finally Section 6 concludes the paper and proposes further research plans.

II. RELATED WORK

In [3] the authors have defined social network sites (SNSs) as web-based services that allow individuals to (1) construct a public or semi-public profile within a bounded system, (2) articulate a list of other users with whom they share a connection, and (3) view and traverse their list of connections and those made by others within the system. The nature and nomenclature of these connections may vary from site to site.

According to this definition, the first recognizable social network site launched in 1997. SixDegrees.com allowed users to create profiles, list their Friends and, beginning in 1998, surf the Friends lists. Each of these features existed in some form before SixDegrees, of course. Profiles existed on most major dating sites and many community sites. AIM and ICQ buddy lists supported lists of Friends, although those Friends were not visible to others. Classmates.com allowed people to affiliate with their high school or college and surf the network for others who were also affiliated, but users could not create profiles or list Friends until years later. SixDegrees was the first to combine these features.

After that social networks have developed rapidly and the number of features increased. Nowadays most sites support the maintenance of pre-existing social networks, but others help strangers connect based on shared interests, political views, or activities. Some sites cater to diverse audiences, while others attract people based on common language or shared racial, sexual, religious, or nationality-based identities. Sites also vary in the extent to which they incorporate new information and communication tools, such as mobile connectivity, blogging, and photo/video-sharing.

As the functions of the SNSs flared, the number of users increased rapidly. Handling the extending number of users efficiently in SNSs is a key issue as it was visible in case of Friendster. Friendster was launched in 2002 as a social complement to Ryze. It was designed to help friends-of-friends meet, based on the assumption that friends-of-friends would make better romantic partners than would strangers. As Friendster's popularity surged, the site encountered technical and social difficulties. Friendster's servers and databases were ill-equipped to handle its rapid growth, and the site faltered regularly, frustrating users who replaced email with Friendster.

Huge amount of papers and popular books, such as Barabási’s Linked [4] study the structure and principles of dynamically evolving large scale networks like the Internet and networks of social interactions. Many features of social processes and the Internet are governed by power law distributions. Following the terminology in [5] a nonnegative random variable X is said to have a power law distribution if $\Pr[X \geq x] \sim cx^{-\alpha}$, for constant $c>0$ and $\alpha>0$. In a power law distribution asymptotically the tails fall according to the power α, which leads to much heavier tails than other common models.

Distributions with an inverse polynomial tail have been first observed in 1897 by Pareto [6] (see. [7]), while describing the distribution of income in the population. In 1935 Zipf [8] and Yule [9] investigated the word frequencies in languages and based on empirical studies he stated that the frequency of the n-th frequent word is proportional to $1/n$.

Mislove et al. [10] studied the graph properties of several online real-world social networks. Their paper presents a large-scale measurement study and analysis of the structure of multiple online social networks. They examined data gathered from four popular online social networks: Flickr, YouTube, LiveJournal, and Orkut. They crawled the publicly accessible user links on each site, obtaining a large portion of each social network's graph. Their data set contains over 11.3 million users and 328 million links. Their measurements show that high link symmetry implies indegree equals outdegree; users tend to receive as many links as the give, the observed networks are power-law with high symmetry.

In [11], the graph structure of the Web has been investigated and it was shown that the distribution of in- and out-degree of the web graph and the size of weekly and strongly connected components are well approximated by power law distributions. Nazir et al. [12] showed that the in- and out-degree distribution of the interaction graph of the studied MySpace applications also follow such distributions.

There has been a great deal of theoretical work on designing random graph models that result in a Web-like graph. Barabási and Albert [13] describe the preferential attachment model, where the graph grows continuously by inserting nodes, where new node establishes a link to an older node with a probability which is proportional to the current degree of the older node. Bollobás et al. [14] analyze this process rigorously and show that the degree distribution of the resulting graph follow a power law. Another model based on a local optimization process is described by Fabrikant et al. [15]. Mitzenmacher [16] gives an excellent survey on the history and generative models for power law distributions. Aiello et al. [17] studies random graphs with power law degree distribution and derives interesting structural properties in such graphs.

The key difference between these researches and our work is that we extended social networks with mobile phone support and we discovered that the distribution of similarities follows power law. We proposed a model to calculate the number of similarities and despite the infinite variance of power law distribution we proved the accuracy of our model.

III. STRUCTURE OF PHONEBOOK-CENTRIC SOCIAL NETWORKS

Phonebook-centric social networks are extending the well-known social network sites, they have a similar web user interface, but they add several major mobile phone related functions to the system. Following consider social networks as graphs. In case of general social networks, nodes are representing registered members and edges between them represent social relationships (e.g. friendship). After this we should notice that each member has a private mobile phone with a phonebook (Figure 1).

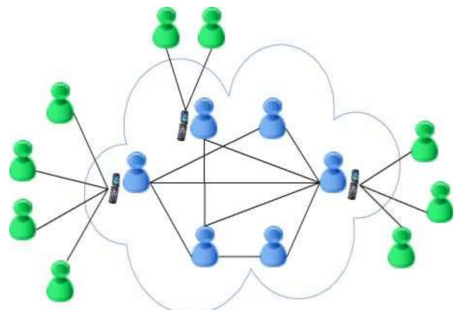

Figure 1. Phonebook-enabled social network

On Figure 1 we can see that phonebook contacts results new type of nodes in the graph representation and the edges between these private phonebook contacts and members represent which member "owns" those private contacts.

One of the key advantages of phonebook centric social networks is that they allow real synchronization between private phonebook contacts and the social network. In order to enable such mechanism we need a similarity detecting algorithm. This algorithm is able to compare two person entries (members and private contacts too) and determine whether they are likely similar, if so, it proposes a probability value to this detected similarity as well. The details of the algorithm are discussed in Section 5.

Figure 2 represents the graph structure if the similarity detecting algorithm has finished comparing the relevant person entries.

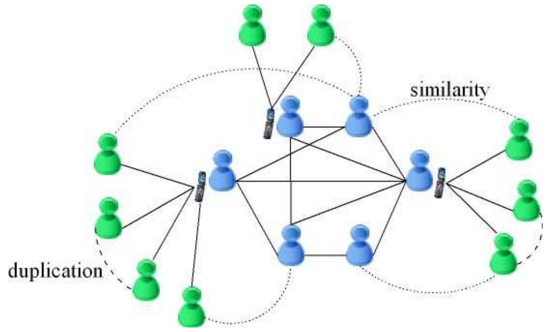

Figure 2. Detected similarities and duplications

On Figure 2 the dotted edges between member and private contacts represent detected similarities and broken lines between two private contacts illustrate possible duplications in the phonebooks. Duplications are detected as a positive side effect of the similarity detecting algorithm.

After similarities and duplications are detected there is a semi-automatic step, the members -who have private contacts in their phonebook, which are detected as similar to other members - have to decide whether detected similarities are relevant ones. In addition to that, members can also decide about the relevancy of detected duplications in their phonebooks. Figure 3 represents the graph structure after some of the members have resolved the detected similarities and duplication.

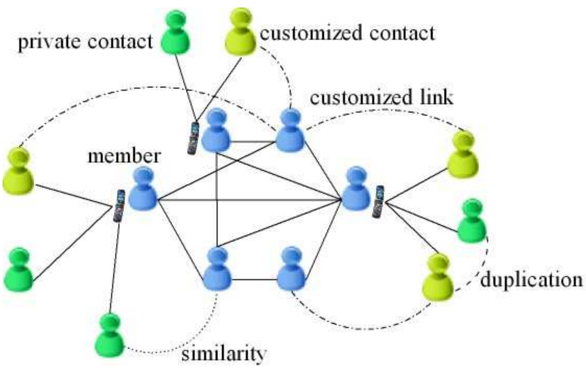

Figure 3. Resolving similarities and duplications

After the resolve, it can be noticed that one of the private contacts of the most left member has been deleted because it was a relevant duplication in the phonebook and the owner member had found it relevant. The other on the right side still remained because that member has not decided about it yet.

Besides that we can see on Figure 3 that four from the five similarities were resolved (members found them relevant) and there is still one in the system (the member has not checked it yet). Resolving a similarity means that a customized link edge is being formed between the private contact(s) in one's phonebook and the relevant member who represent the same person in the system. The private contacts that are linked to members via this type of customized links are called *customized contacts*. One of the key advantages of phonebook-centric social networks are these customized links, because if a member changes his personal detail on the web user interface (adds a new phone number, uploads a new image, changes the website address, etc) it will be automatically propagated to those phonebooks where there is a customized contact related to this member. Additional important advantages of phonebook-centric social networks are:

- Private contacts can be managed (list, view, edit, call, etc.) from a browser.
- Similarity detecting algorithm realizes the user if duplicate contacts are detected in its phonebook and warns about it.
- Private contacts are safely backed up in case the phone gets lost.
- Private contacts can be easily transferred to a new phone if the user replaces the old one.
- Phonebooks can be shared between multiple phones, if one happens to use more than one phone.
- It is not necessary to explicitly search for the friends in the service, because it notices if there are members similar to the private contacts in the phonebooks and warns about it.

The detailed structure and edge rule definition was described in [18].

IV. Number of Similarities

We model the number of similarities generated during a member registration by a probability variable *X*. More precisely, *X* models the number of similarities proposed by the automatic similarity detection algorithm. In [19] we showed how to estimate the total number of similarities in the system. Following we summarize this model.

The total number of accepted similarities N_S in a phonebook-centric social network can be estimated with the following formula:

$$N_S = N_M E[X] P_R, \tag{3}$$

where N_M is the number of registered members and P_R is the rate of the similarities accepted by the users. Measurements in [19] showed that P_R can be approximated with *0.9*. In order to calculate *E[X]*, we need the probabilities *Pr[X=x]*, which can be obtained from the complementary cumulative distribution function $\Pr[X \geq x] \sim cx^{-\alpha}$ by derivation:

$$\Pr[X = x] \sim c' x^{-(\alpha+1)}. \tag{4}$$

In order to be a probability distribution, $\sum_{x=1}^{\infty} c' x^{-(\alpha+1)} = 1$. Note, that *x* starts from one, because a new member registration involves at least one similarity, because the system allows registration only by invitation. Ttherefore, the new member is already in the phonebook of the inviting member. Thus, $c' = 1/\zeta(\alpha+1)$, where $\zeta(.)$ denotes the Riemann Zeta function. Then the expected value is:

$$\begin{aligned} E[X] &= \sum_{x=1}^{\infty} x \Pr[X = x] \\ &= \sum_{x=1}^{\infty} x \frac{1}{\varsigma(\alpha+1)} x^{-(\alpha+1)} \\ &= \frac{1}{\varsigma(\alpha+1)} \sum_{x=1}^{\infty} x^{-\alpha} = \frac{\varsigma(\alpha)}{\varsigma(\alpha+1)}. \end{aligned} \tag{5}$$

The expected total number of accepted similarities N_S in a phonebook-centric social network can be estimated with the following formula:

$$N_S = N_M \frac{\varsigma(\alpha)}{\varsigma(\alpha+1)} P_R. \tag{6}$$

For $\alpha>1$, $\varsigma(\alpha)/\varsigma(\alpha+1)$ is a finite constant. In our case, for $\alpha=1.276$, we obtain that the expected total number of similarities is

$$N_S = 2.9196 * 420 * 0.9 = 1103. \tag{7}$$

However in this model the *X* random probability value has power law distribution which has infinite variance thus the accuracy of this model is an issue. In the next section we show how to prove the accuracy of this model by stating and a general theorem related to the variance of power law distributions with relevant upper bound.

V. Variance Model for Power Law Distribution with Upper Bound

Power law distribution has infinite variance, which prevents to apply the central limit theorem in order to obtain that the total number of similarities will be close to their expected value. However we can use the following fact

Fact: If the phonebooks do not contain duplicates then the number of similarities caused by a member is at most $2(N_M-1)$ [19].

With other words, in the interval $[0,2(N_M-1)]$ the distribution of similarities follows a power law and the probability of higher similarities is zero. In order to see this, note that a member *u* can be similar to at most one private contact of each of the other N_M-1 members and, for each private contact of *u*, there is at most one similar member in the network.

We show that the distribution of similarities resulting from this fact has a finite variance. This allows us to use the central limit theorem to prove the accuracy of our estimation of the total number of similarities in Section 4.

Theorem: Let *X* be a random variable with $\Pr[X = x] = cx^{-\beta}$ if $x \leq n$ and $\Pr[X = x] = 0$ otherwise, where $\beta=\alpha+1$, $2<\beta<3$. In this case the variance can be estimated with $\sigma^2 X = \Theta(n^{3-\beta})$.

For the proof we used two lemmata.

Lemma 1: Let X be a random variable with $\Pr[X=x]=cx^{-\beta}$ if $x \le n$ and $\Pr[X=x]=0$ otherwise, where $\beta=\alpha+1$, $2<\beta<3$. In this case the variance is $\sigma^2 X = \mathrm{O}\left(n^{3-\beta}\right)$.

From the Steiner formula, the variance is calculated as $\sigma^2 X = E[X^2]-(E[X])^2$. $E[X]$ was defined previously, thus we need to calculate only the $E[X^2]$. By definition:

$$E[X^2]=\sum_{x=1}^{\infty} x^2 \Pr[X=x] = \sum_{x=1}^{\infty} x^2 c \frac{1}{x^\beta}. \tag{8}$$

Now we can apply that n is an upper bound on the value of X. This way (1) can be followed as:

$$E[X^2]=c\sum_{x=1}^{n} x^{2-\beta}.$$

Let $y=\frac{1}{c}E[X^2]$. (9)

Following we show an upper estimation for y. In order to do so we create an upper model for the function of y by using the powers of 1/2. Let $z=2^{\frac{1}{\beta-2}}$, then

$$z^{2i}\Pr[X=z^i]=(z^i)^{2-\beta}=\left(2^{\frac{1}{\beta-2}i}\right)^{2-\beta}=2^{-i} \tag{10}$$

Now we are able to approximate y from top:

$$\begin{aligned} y &\le \sum_{i=0}^{\log_z n}\left(z^{i+1}-z^i\right)\left(z^i\right)^{2-\beta} \\ &= \sum_{i=0}^{\log_z n}\left(z^{i+1}-z^i\right)2^{-i} \\ &= \sum_{i=0}^{\log_z n}(z-1)z^i 2^{-i} \\ &= (z-1)\sum_{i=0}^{\log_z n}\left(\frac{z}{2}\right)^i \\ &= (z-1)\left(\frac{\left(\frac{z}{2}\right)^{\log_z n+1}-1}{\frac{z}{2}-1}\right) \\ &= (z-1)\left(\frac{\frac{z}{2}n^{\frac{1}{\log_{z/2} z}}-1}{\frac{z}{2}-1}\right) \\ &= (z-1)\left(\frac{\frac{z}{2}n^{\frac{1}{1+\log_{z/2} 2}}-1}{\frac{z}{2}-1}\right). \end{aligned} \tag{12}$$

The explanation to the last step:

$$\log_{z/2} z = \log_{z/2} 2\frac{z}{2} = 1+\log_{z/2} 2$$

To continue, first we have to check the following calculation. Remember that z was described with β and $\beta=\alpha+1$. This way:

$$\log_{z/2} 2 = \frac{\log_2 2}{\log_2 z/2} = \frac{1}{\log_2\left(\frac{2^{\frac{1}{\beta-2}}}{2}\right)} = \frac{1}{\frac{1}{\beta-2}-1} = \frac{\beta-2}{3-\beta}. \tag{13}$$

Therefore:

$$n^{\frac{1}{1+\log_{z/2} 2}} = n^{\frac{1}{1+\frac{\beta-2}{3-\beta}}} = n^{3-\beta}. \tag{14}$$

This way (2) looks as follows:

$$y \le (z-1)\left(\frac{\frac{z}{2}n^{\beta-3}-1}{\frac{z}{2}-1}\right). \tag{15}$$

Next we show that the variance by applying the Steiner formula and the previous calculations is $O(n^{3-\beta})$:

$$
\begin{aligned}
\sigma^2 X &= E[X^2] - (E[X])^2 = cy - \Theta(1) \\
&= c(z-1)\left(\frac{\frac{z}{2} n^{\beta-3} - 1}{\frac{z}{2} - 1} \right) - \Theta(1) \\
&\le c\left(\frac{(z-1)z}{z-2} n^{3-\beta} \right) - \Theta(1) \\
&= O\left(n^{3-\beta}\right).
\end{aligned} \tag{16}
$$

□

Lemma 2: Let X be a random variable with $\Pr[X = x] = cx^{-\beta}$ if $x \le n$ and $\Pr[X = x] = 0$ otherwise, where $\beta = \alpha + 1$, $2 < \beta < 3$. In this case the variance is $\sigma^2 X = \Omega\left(n^{3-\beta}\right)$.

Similarly to *Lemma 1* if we approximate by using:

$$y \ge \sum_{i=0}^{\log_z n} \left(z^{i+1} - z^{i}\right) 2^{-(i+1)}, \tag{17}$$

we can see that $\sigma^2 X = \Omega\left(n^{3-\beta}\right)$.□

Proof of Theorem is straightforward by applying Lemma 1 and 2:

$\sigma^2 X = \Theta\left(n^{3-\beta}\right)$, because $\sigma^2 X = \Omega\left(n^{3-\beta}\right)$ and $\sigma^2 X = O\left(n^{3-\beta}\right)$ □

In our case the upper bound n to the total number of similarities is $2(N_M - 1)$.

VI. Conclusion and Future Work

Social network sites are becoming more and more important in everyday life. Phonebook-centric social networks enable to manage online and mobile relationships within one system. The key mechanism of such networks is a similarity handling algorithm which detects similarities between members of the network and phonebook entries.

The number of similarities is a key parameter from scalability point of view. Previously we showed how to estimate the expected number of similarities. The distribution of similarities can be modeled with a random probability variable X with $\Pr[X \ge x] \sim cx^{-\alpha}$, if $x \le n$ and $\Pr[X \ge x] = 0$ otherwise, where $\alpha > 1$.

In this paper we showed that using this model, the distribution of similarities has a finite variance. This allows us to use the central limit theorem to prove the accuracy of our estimation of the total number of similarities. This model can be used generally in other similar distributions.

Future work includes further analysis of Phonebookmark database and creating a central limit theory based model which proves that the probability of having different amount of similarities in the system than the result of the model is very low.

Acknowledgment

This work is supported by the New Hungary Development Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002 and TAMOP 4.2.1/B-09/1/KMR-2010-0003).

References

[1] Alexa. http://www.alexa.com/topsites. February 2010.

[2] Facebook statistics, http://www.facebook.com/press/info.php?statistics, February, 2010.

[3] D. M. Boyd, N. B. Ellison, *Social network sites: Definition, history, and scholarship*, Journal of Computer-Mediated Communication, Volume 13, Issue 1 (2007)

[4] A.-L. Barabási, R. Albert. *Emergence and scaling in random networks*. *Science*, Vol. 286, paged: 509-512, 1999.

[5] A. Fabrikant, E. Koutsoupias, and C. H. Papadimitriou. *Heuristically Optimized Trade-offs: A New Paradigm for Power Laws in the Internet.* In *Proc. of ICALP*, pages: 110-122, 2002.

[6] V. Pareto. *Course d'economie politique professé à l'université de Lausanne*, 3 volumes, 1896-7.

[7] M. Mitzenmacher. *A brief history of generative models for power law and lognormal distributions*. *Internet Mathematics*, Vol. 1, pages: 225-251, 2001.

[8] G. K. Zipf. *The Psycho-Biology of Language. An Introduction to Dynamic Philology*. Houghton Mifflin, Boston, MA, 1935.

[9] G. U. Yule. *Statistical study of literary vocabulary*, Cambridge University Press, 1944.

[10] A. Mislove, M. Marcon, K. P. Gummadi, P. Druschel, and B. Bhattacharjee. *Measurement and analysis of online social networks*. In ACM/USENIX IMC, 2007.

[11] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, A. Tomkins, and J. Wiener. *Graph structure in the web*. In Proc. of the 9th international World Wide Web conference on Computer networks, 2000.

[12] Nazir, S. Raza and C.-N. Chuah. *Unveiling Facebook: A measurement Study of Social Network Based Applications*. In: Proc. ACM Internet Measurement Conference (IMC), 2008.

[13] A.-L. Barabási, R. Albert. Emergence and scaling in random networks. *Science*, Vol. 286, 509-512, 1999.

[14] B. Bollobás, O. Riordan, J. Spencer, G. Tusnady. The degree sequence of a scale-free random graph process. *Random Structures and Algorithms*, Vol. 18(3), 279-290, 2001.

[15] A. Fabrikant, E. Koutsoupias, and C. H. Papadimitriou. Heuristically Optimized Trade-offs: A New Paradigm for Power Laws in the Internet. In: *Proc. 29th International Colloquium on Automata, Languages and Programming (ICALP)*, 110-122, 2002.

[16] M. Mitzenmacher. A brief history of generative models for power law and lognormal distributions. *Internet Mathematics*, Vol. 1, 225-251, 2001.

[17] W. Aiello, F. R. K. Chung, L. Lu. A random graph model for massive graphs. In: *Proc. 32nd Symposium on Theory of Computing STOC*, 171-180, 2000.

[18] P. Ekler, T. Lukovszki. *Similarity Distribution in Phonebook-centric Social Networks*. In: 5th International Conference on Wireless and Mobile Communications (ICWMC). 2009.

[19] P. Ekler, T. Lukovszki. *Experiences with phonebook-centric social networks*. In: CCNC'10, Las Vegas. 2010.