Asynchronous Filling by Myopic Luminous Robots

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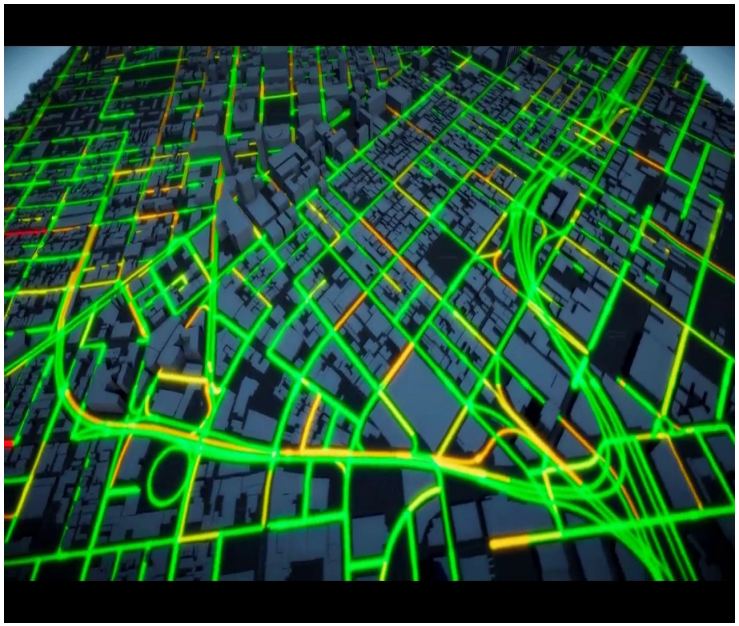
FILLING, UNIFORM DISPERSING

- n robots
- Area:
  - represented by a graph of n vertices
  - unknown
  - connected
  - For each vertex, the adjacent vertices are arranged in a fixed cyclic order
- Robots enter at “Door” vertices
- Robots can move to neighboring vertices
- Robots have to occupy all vertices
- Collision must be prevented

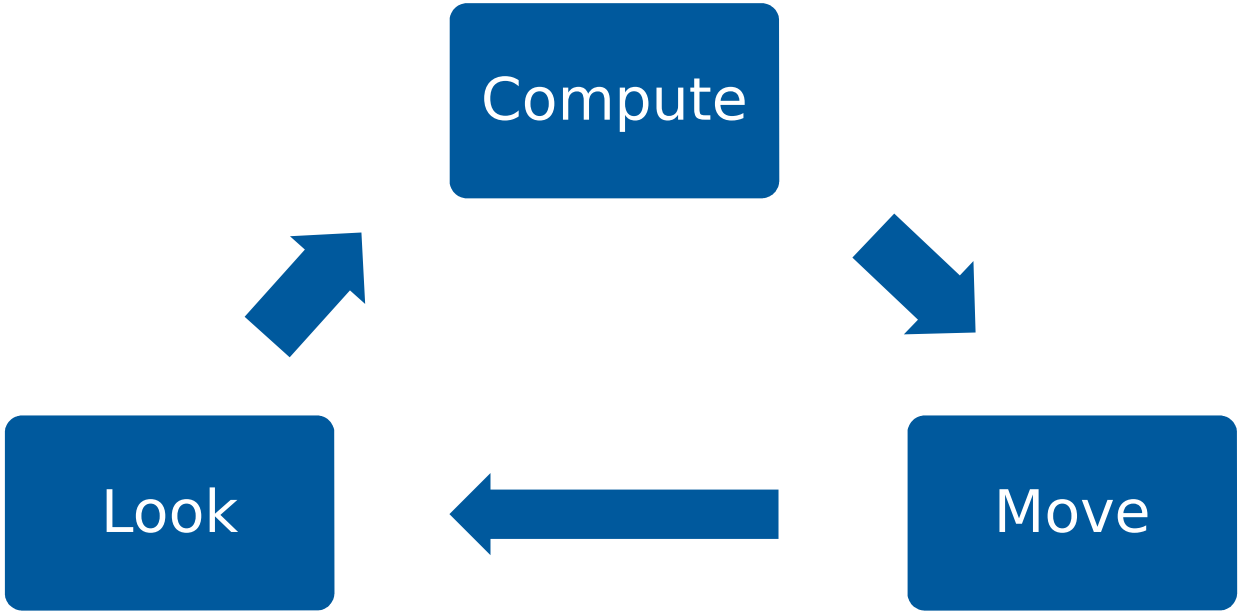


ROBOTS

- Robots have restricted capabilities
  - homogeneus
  - anonymous
  - limited viewig range
  - limited memory
  - no explicit communication
  - visible lights
  - asynchronous

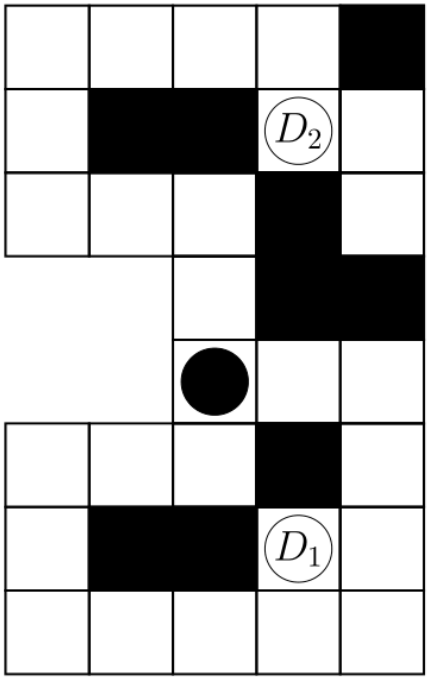
https://ssr.seas.harvard.edu/

LOOK-COMPUTE-MOVE CYCLES



FILLING, DISPERSING

- Orthogonal



FILLING, DISPERSING

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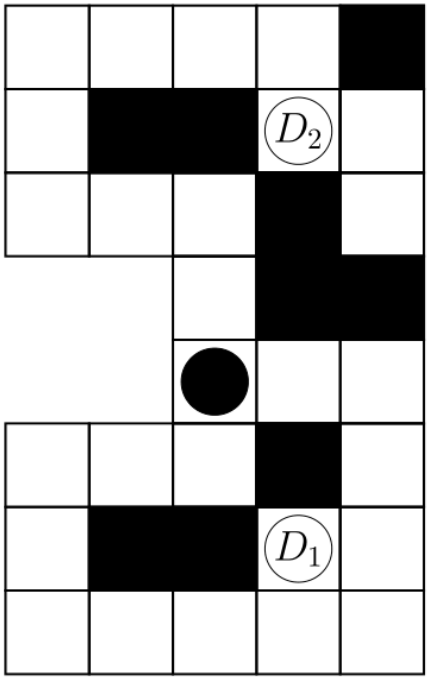
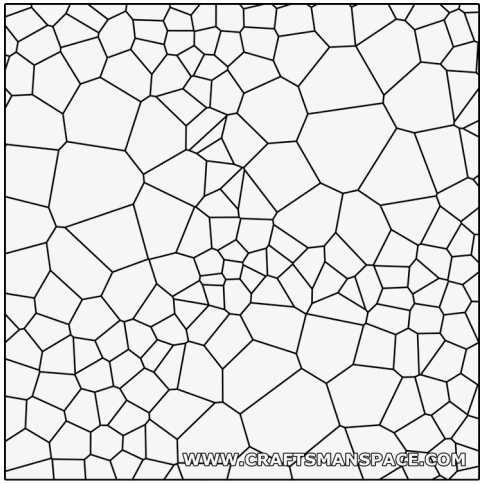


Arbitrary Graph



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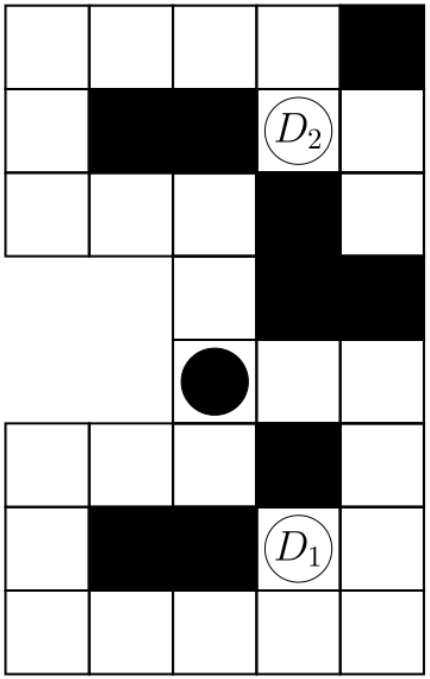

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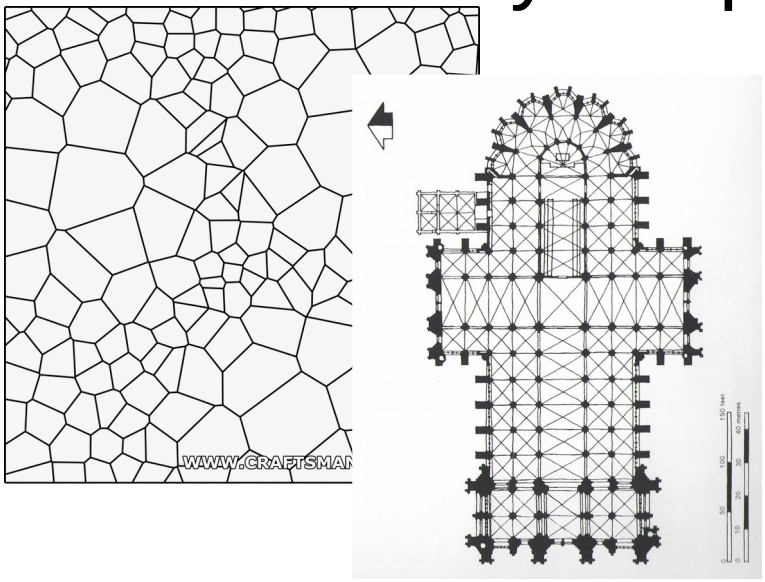

FILLING, DISPERSING

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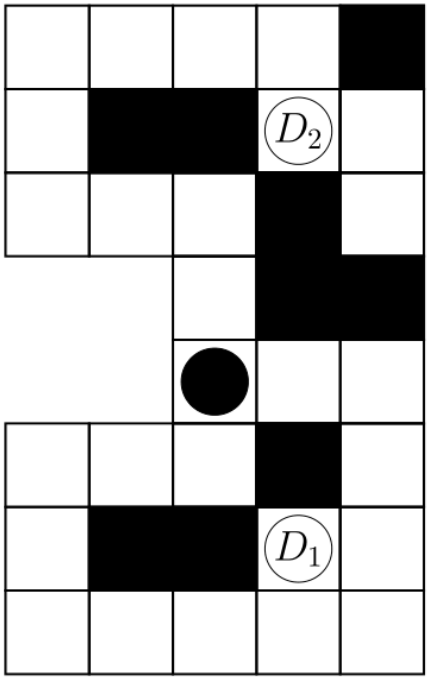

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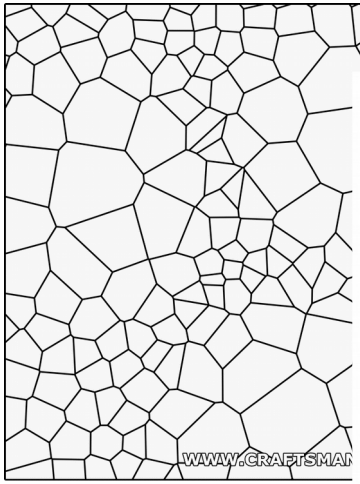

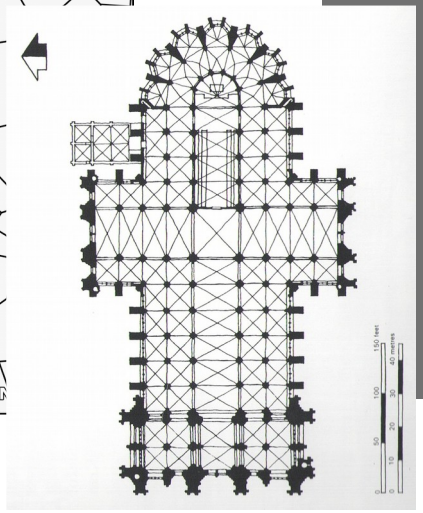

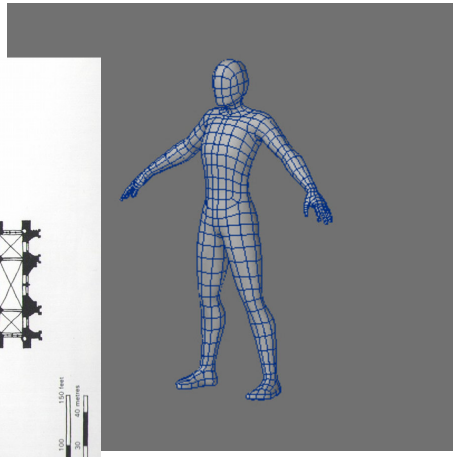

FILLING, DISPERSING

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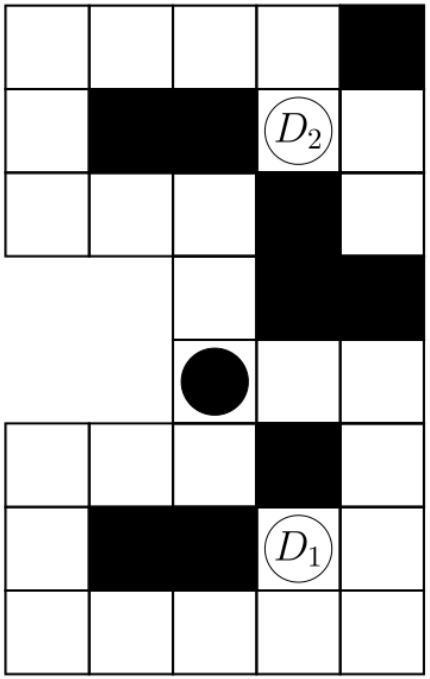

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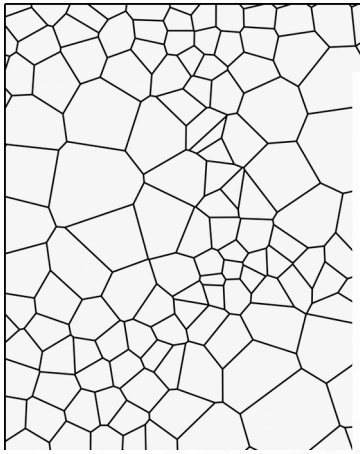

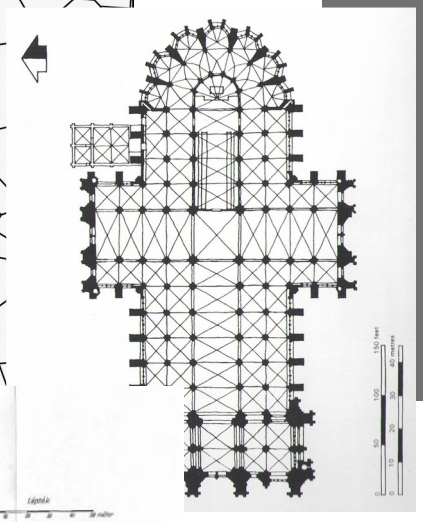

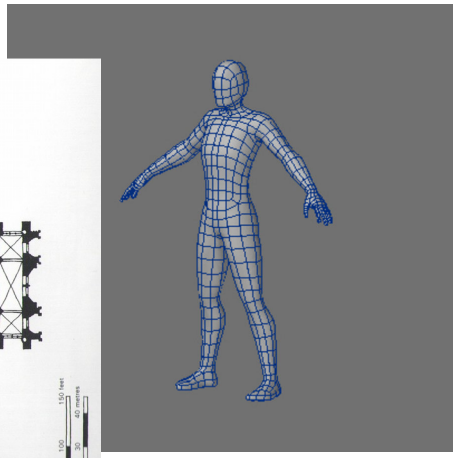

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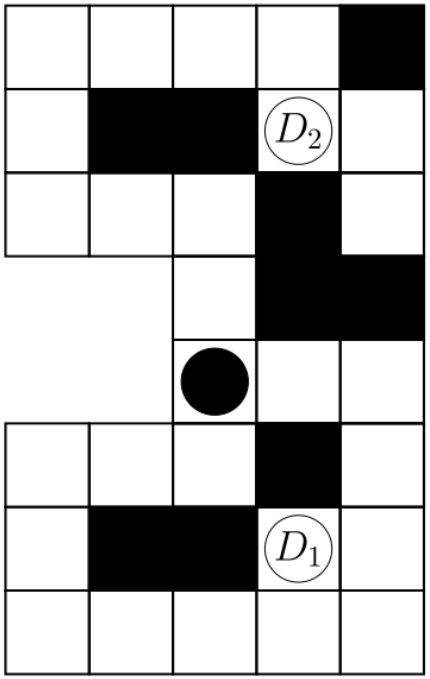

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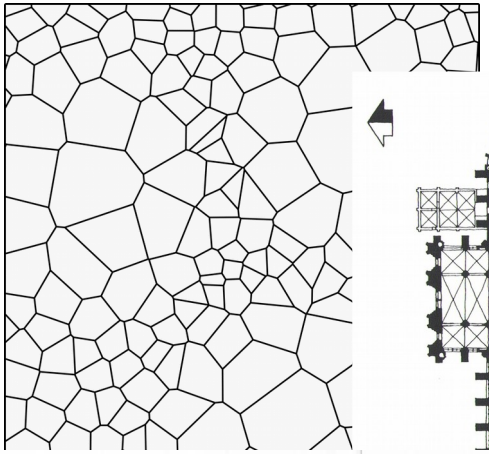
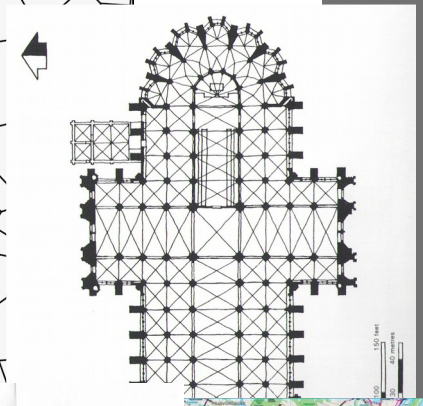
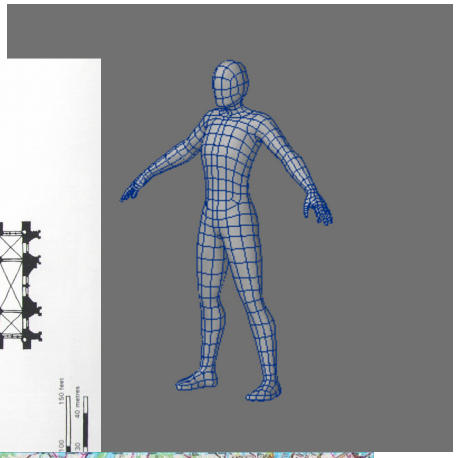
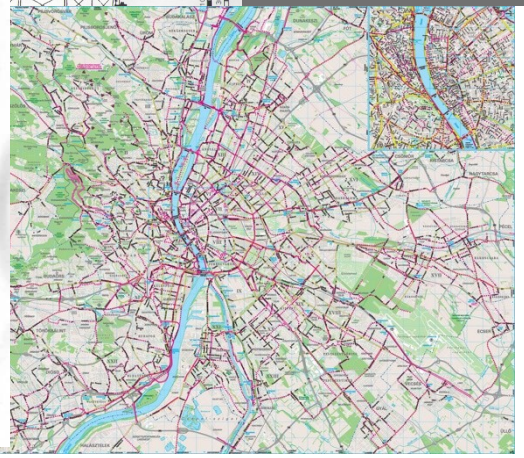






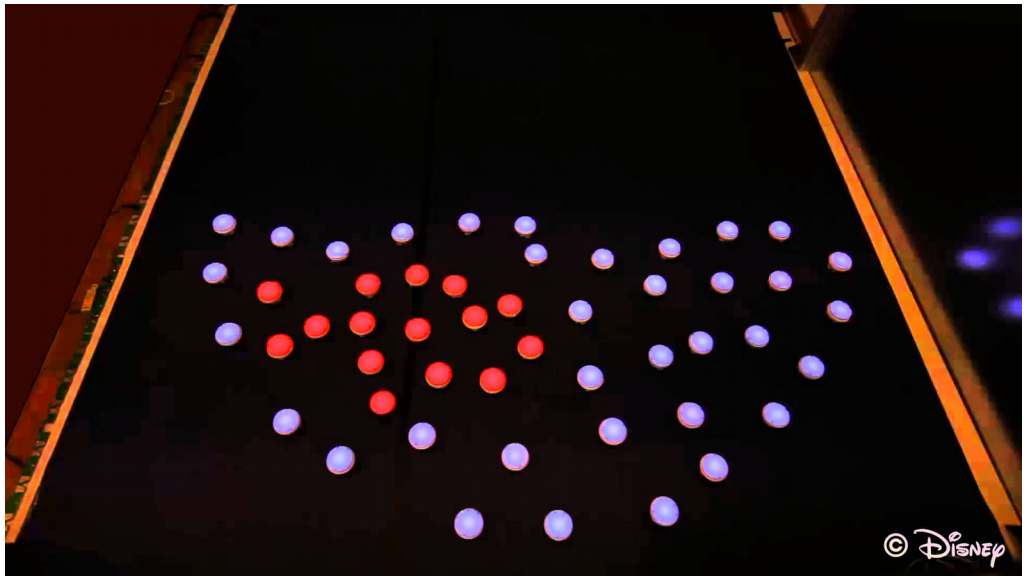


STATE OF THE ART – COLLISIONLESS DISPERSION

Method	FSYNC/ ASYNC	Doors	Viewing range	Comm. range	Memory bits	Area (Orthogonal/ Arbitrary)
BFLF, DFLF [Hsiang et al. 2004]	FSYNC	Single	2	2	2	O
TALK [Barrameda et al. 2013]	ASYNC	Single	2	2	4	O
MUTE [Barrameda et al. 2013]	ASYNC	Single	6	-	9	O
MULTIPLE [Barrameda et al. 2008]	ASYNC	Multiple	3	- k colors	4	O
Single Door [Hideg, Lukovszki 2017]	FSYNC	Single	1	-	13	O
Multiple Door [Hideg, Lukovszki 2017]	FSYNC	Multiple	1	-	13	O
VCM [Hideg, Lukovszki 2018]	FSYNC	Single	1	-	$O(\Delta)$	A
MD-VCM [Hideg, Lukovszki 2018]	FSYNC	Multiple	1	-	$O(\Delta \cdot \log k)$	A

LUMINOUS ROBOTS

- Robots are enhanced with VISIBLE LIGHTS
- that can change color
- Model [Peleg 2005]



Pixelbots, Disney & ETH Zürich

- FSYNC $\not\subseteq$ ASYNC$^{O(1)}$ and ASYNC$^{O(1)}$ $\not\subseteq$ FSYNC [D'Emidio et al. 2016]

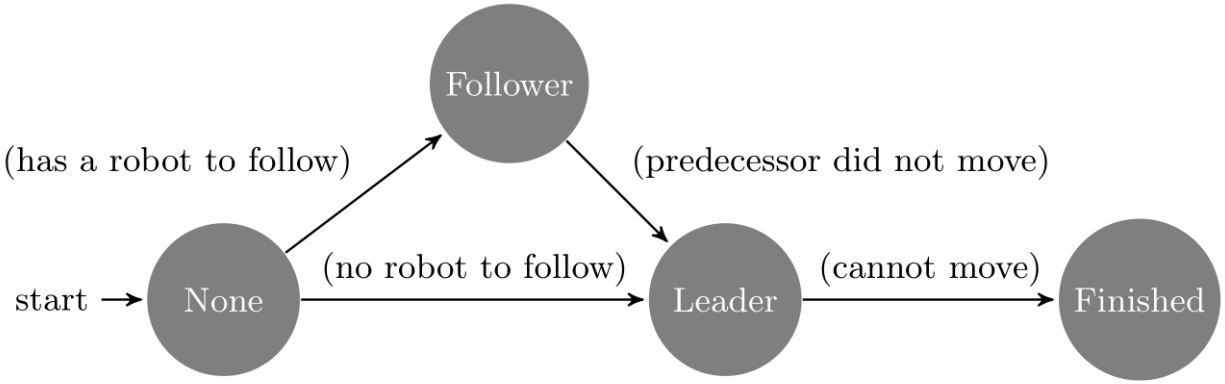
OUR CONTRIBUTION

Method	FSYNC/ ASYNC	Doors	Viewing range	Runtime #async rounds	Persistent memory bits	Colors	Area (Orthogonal/ Arbitrary)
PACK	ASYNC	Single	1	$O(n^2)$	$O(\log \Delta)$	$\Delta+4$	A
Mod-PACK	ASYNC	Single	1	$O(n^2 \log \Delta)$	$O(\log \Delta)$	$O(1)$	A
BLOCK	ASYNC	Single	2	$O(n)$	$O(\log \Delta)$	$\Delta+4$	A
k-Door-BLOCK	ASYNC	Multiple	2	$O(n)$	$O(\log(\Delta+k))$	$\Delta+k+4$	A

First asymptotic bounds for filling in the ASYNC model.
Only termination in finite time has been proven in previous works.

VISIBILITY: 1-HOP – PACK ALGORITHM

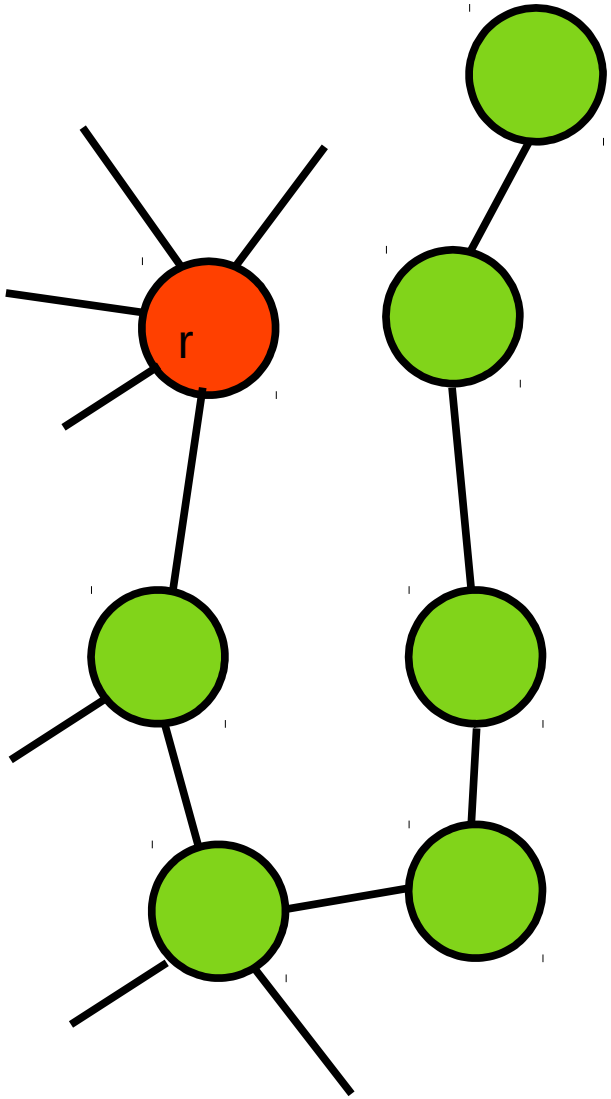
- Follow-the-Leader basic concept:



- Mimics a DFS traversal of the unknown graph
- Virtual Chain: Path of the current Leader from the Door
  - All not “Finished” robots are on the virtual chain
- Tasks to solve:
  - Prevent collision

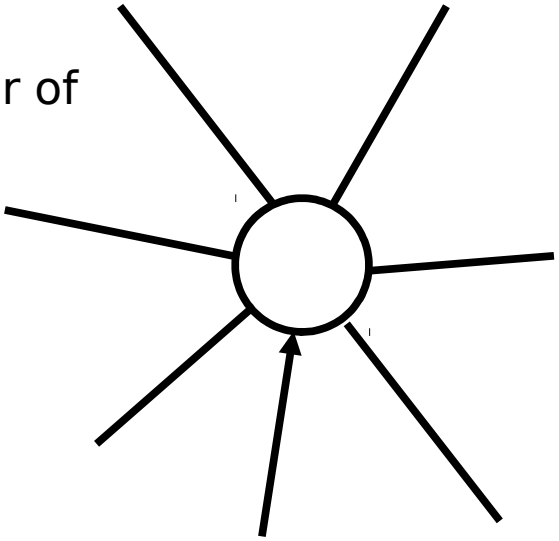
VISIBILITY: 1-HOP – PACK ALGORITHM

- Leader moves to unvisited vertices
- Packed state:
  Each Follower is immediately behind its predecessor
  - All vertices of the chain are occupied by a robot
- Leader only moves, when packed state is reached
  - No other robot can move in this state
  - Collision-freeness



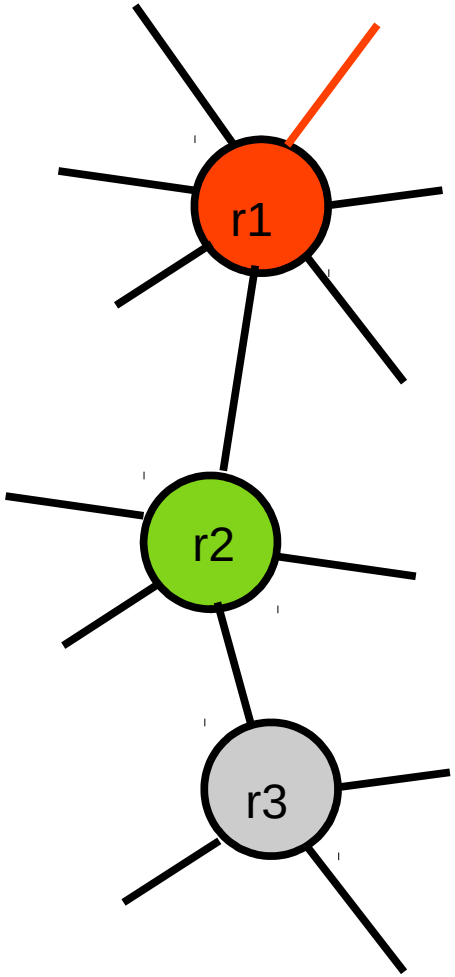
VISIBILITY: 1-HOP – PACK ALGORITHM

- Δ+4 colors:
  - Δ colors (DIR) indicating the direction of the target vertex (relative to the entry direction in cyclic order of neighbors)
  - 2 colors (CONF, CONF2) for confirmation
    - Robot can only move if the successor is behind it and the DIR color is confirmed
  - 1 color (MOV) during movement
  - Light is off (considered as color)
- Leader moves to an unoccupied neighboring vertex – if exists.
- If there is no unoccupied neighboring vertex, then
  - the Leader switches to Finished state and
  - the successor becomes the new Leader ”Taking the Leadership”



VISIBILITY: 1-HOP – PACK ALGORITHM

- Leader:
  - Can only move to an unvisited vertex. When it wants to move, it
  - shows the direction: setting the DIR color, and
  - it waits until its successor allows to move by setting its CONF color. During the movement, the Leader shows the MOV color.
  - When its successor sets CONF color, the chain is in Packed state.



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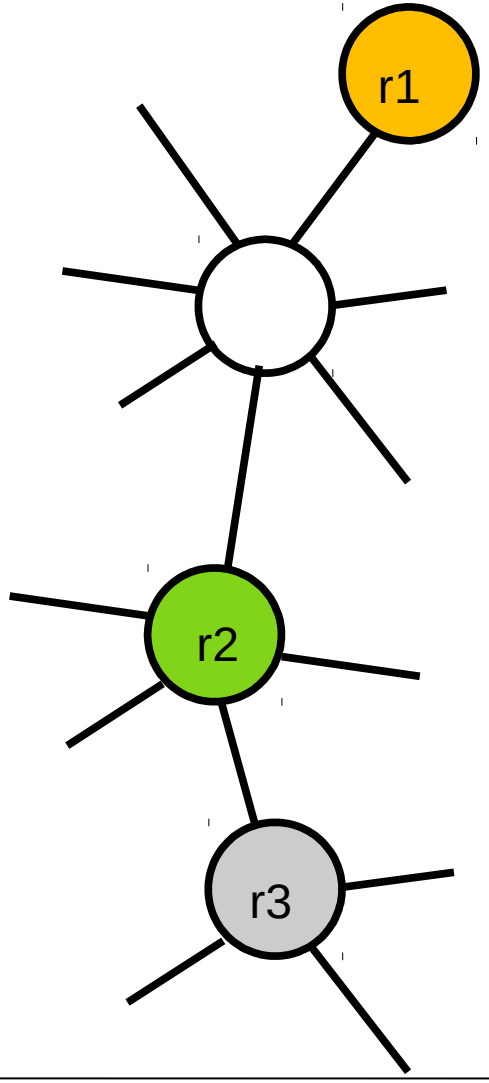
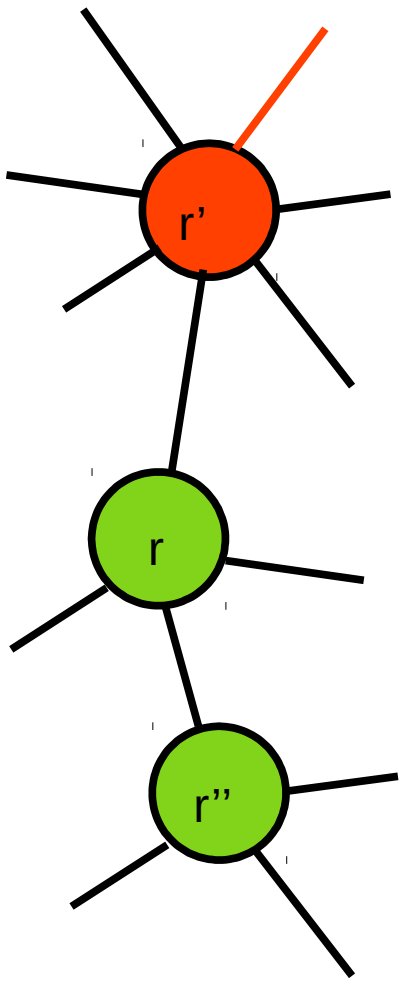


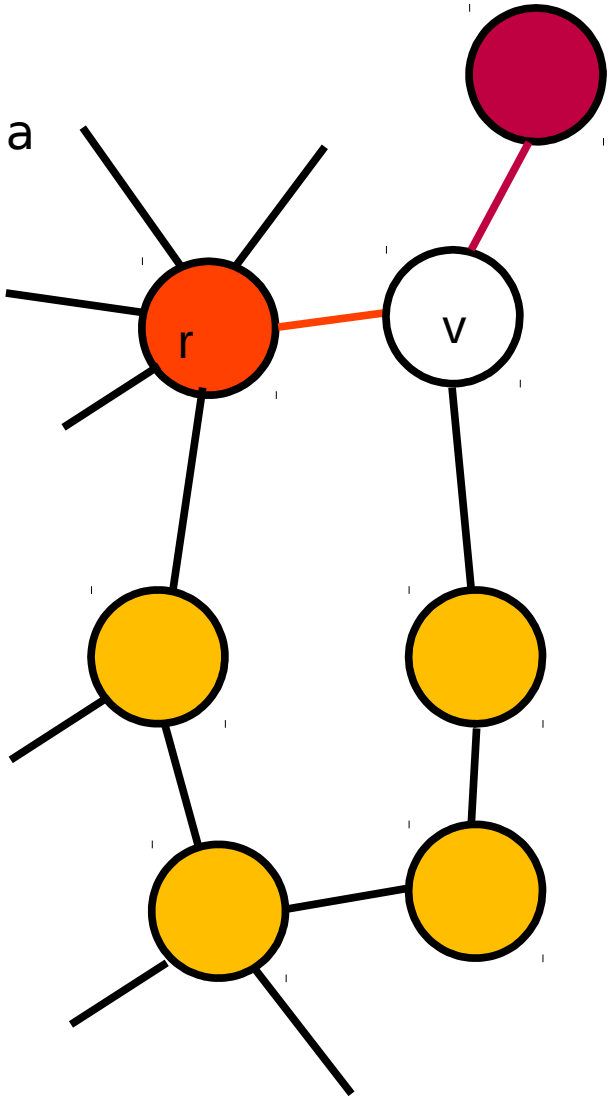
VISIBILITY: 1-HOP – PACK ALGORITHM

- Follower: Follows its predecessor.
  - Follower r sets the CONF color if and only if
    i) the predecessor of r is showing its direction, and
    ii) the successor r'' of r – if exists – have set its CONF color (i.e. the successor knows in which direction r will move).
  - This allows the predecessor r' of r to move to its destination knowing:
    i) all the robots behind r' have set CONF color, and
    ii) the robots behind r' will not move until r' moved.
  - When r' is the Leader, the chain is in Packed state.



VISIBILITY: 1-HOP – PACK ALGORITHM

- Leader target change:
  - It might happen that the Leader r chooses a target v, which is unoccupied when r performs its Look operation.



VISIBILITY: 1-HOP – PACK ALGORITHM

- Leader target change:
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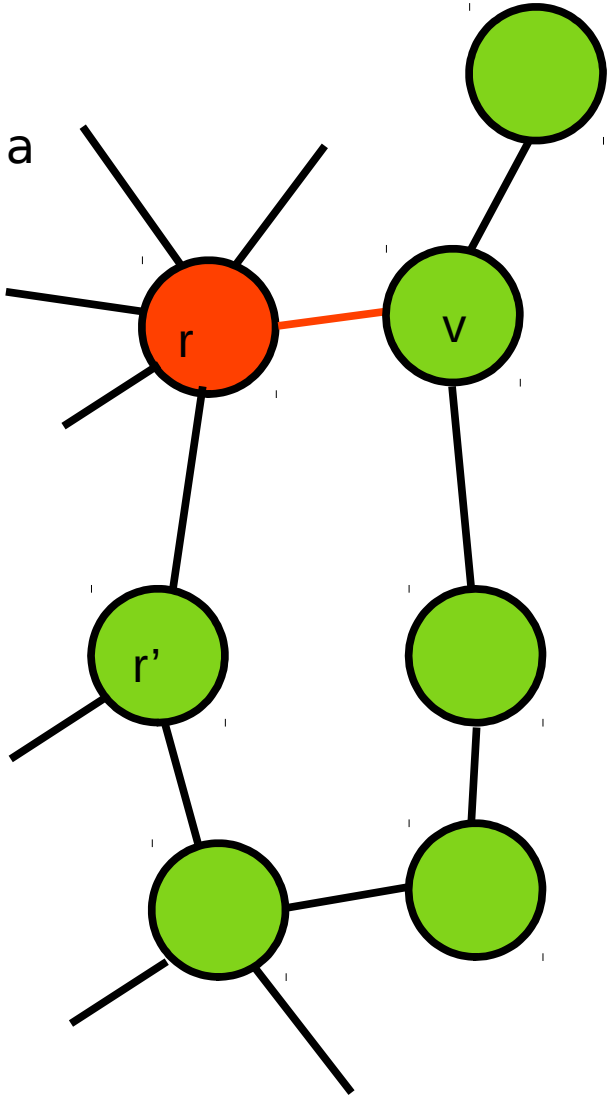
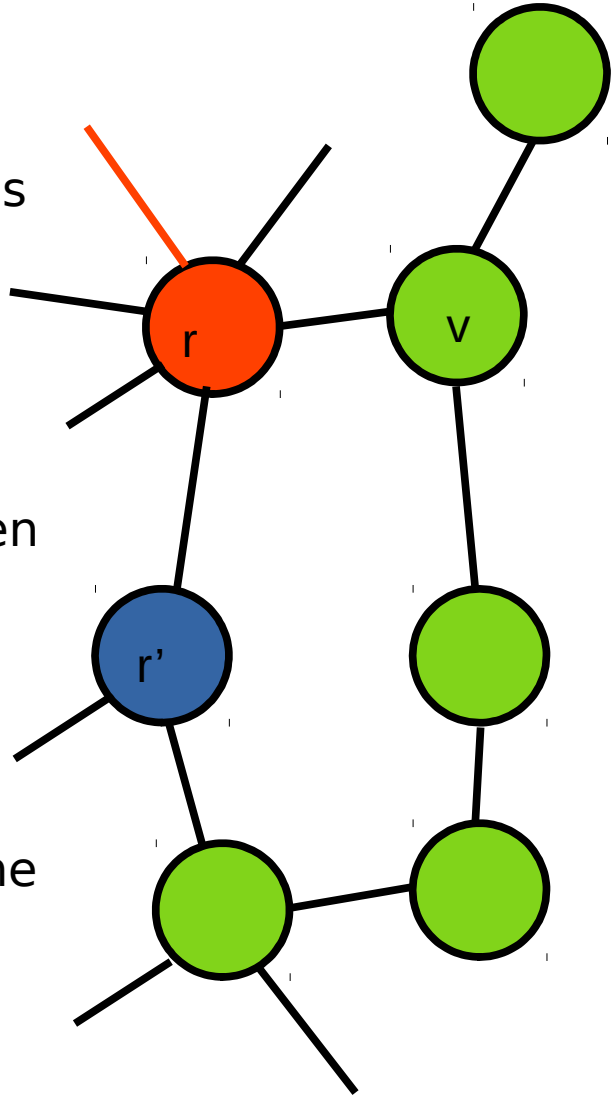
  - When the successor r’ of r sets the CONF color, another robot already moved to v.



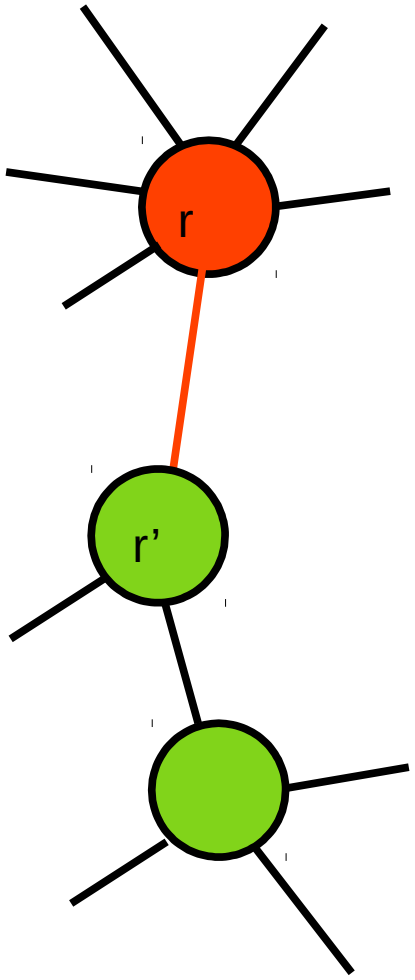
VISIBILITY: 1-HOP – PACK ALGORITHM

- Leader target change:
  - It might happen that the Leader r chooses a target v, which is unoccupied when r performs its Look operation.
  - When the successor r’ of r sets the CONF color, another robot already moved to v.
  - If r has an unoccupied neighboring vertex then
    - r sets the new DIR color and waits until its successor sets the CONF2 color.
    - r moves to the target.
  - Otherwise, r switches to Finished state and the Leadership is taken by r’



VISIBILITY: 1-HOP – PACK ALGORITHM

- Taking the Leadership:
  - When the Leader r cannot move anymore, its successor has to become the new Leader.
  - r sets its DIR color to Δ.
    - color Δ indicates that the Leader cannot move anymore and wants to switch to Finished state, and the leadership must be taken by its successor.
  - successor r’ of r sets its CONF color, waits for the previous Leader to turn off its light.
  - Then r’ becomes the Leader.
    r’ tries to move to an unvisited vertex.



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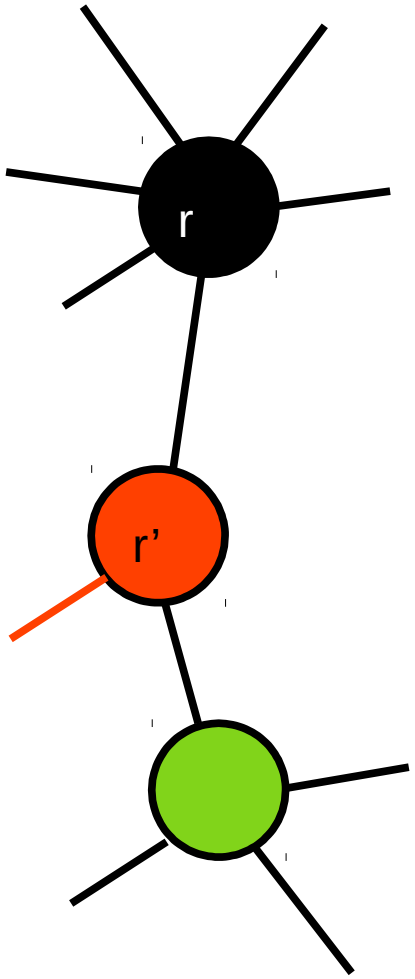


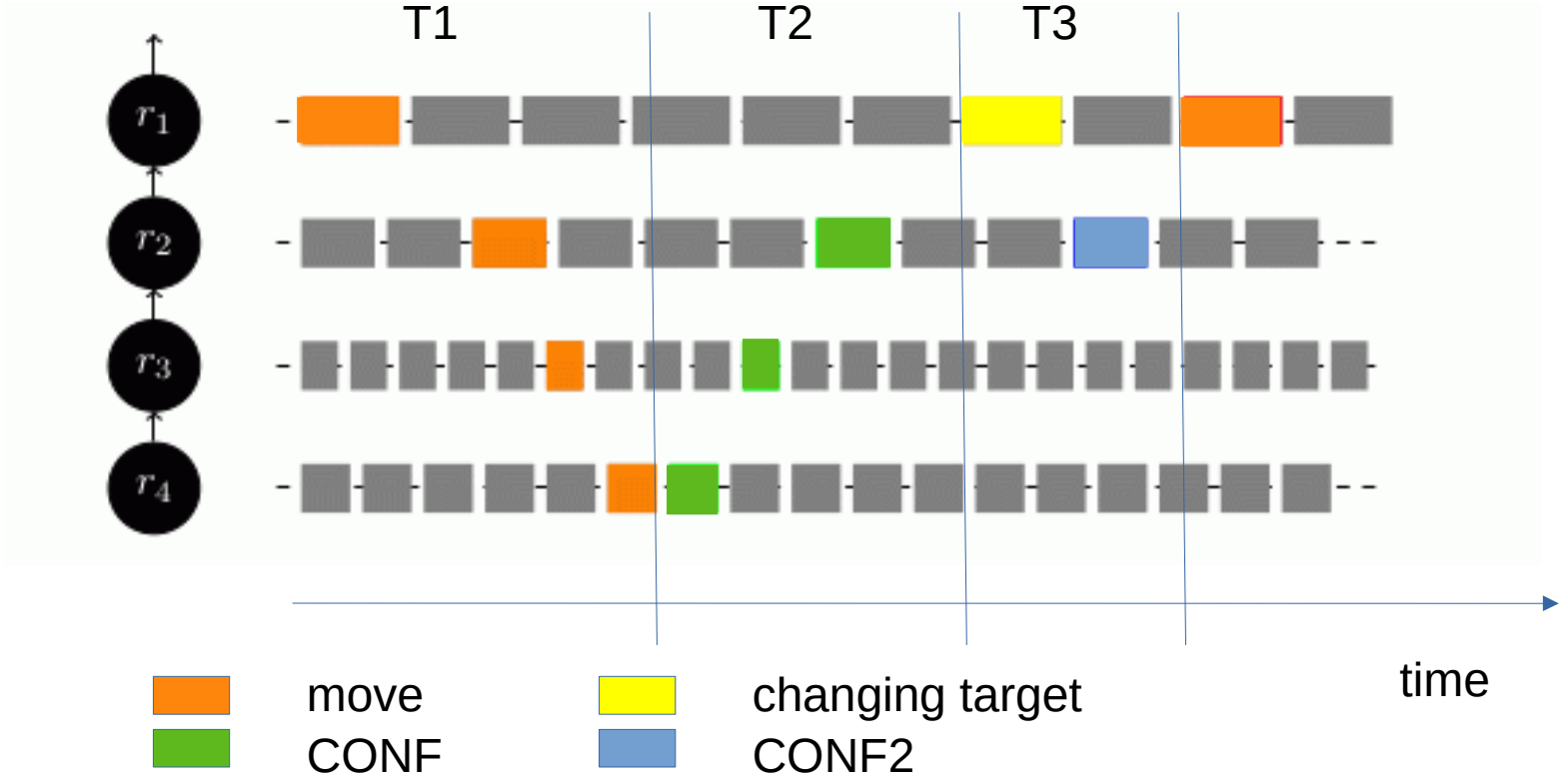
PACK ANALYSIS

- **Theorem 1**. Algorithm PACK fills a connected graph in the ASYNC model by robots having a
  - visibility range of 1 hop,
  - $O(\log \Delta)$ bits of persistent storage, and
  - $\Delta + 4$ colors, including the color when the light is off.
  - PACK runs in $O(n^2)$ asynchronous rounds.

PACK – ANALYSIS

Proof idea for the running time:
T : time until occupying a new unvisited vertex
a) Leader has an unoccupied neighboring vertex

$T = T1 + T2 + T3 \leq 2i + 2 \leq 2n$ rounds



PACK – ANALYSIS

Proof idea for the running time:

b) Leader has no unoccupied neighboring vertex
Taking the leadership : ≤ 5 rounds

Each vertex can take the leadership only once.
Time for all leadership taking: ≤ 5 n rounds.

Overall time for PACK: ≤ n (2n + 5) = $O(n^2)$ rounds.

O(1) COLORS – MOD-PACK ALGORITHM

- Idea:
  - Encode the $L = \Delta + 4$ colors by a sequence of $\lceil \log L \rceil$ bits and
  - transmit this sequence by emulating the Alternating Bit Protocol (ABP), also referred to as Stop-and-wait ARQ

- **Theorem 2:** The modified PACK algorithm fills a connected graph in the ASYNC model by robots having a
  - visibility range of 1 hop,
  - $O(\log \Delta)$ bits of persistent storage, and
  - $O(1)$ colors.
  - The algorithm needs $O(n^2 \log \Delta)$ asynchronous rounds.

2-HOP VISIBILITY – BLOCK ALGORITHM

- Idea:
  - The Leader r sees all robots, that could move to the same target
  - The Leader only chooses a vertex v as the target, if the 1 hop neighborhood of v does not contain any other robot with the light turned on
    - except when the light showing direction Δ (wants to switch to Finished state)
  - A vertex neighboring to a robot with its light on (except the color Δ) is considered as blocked vertex for the Leader.

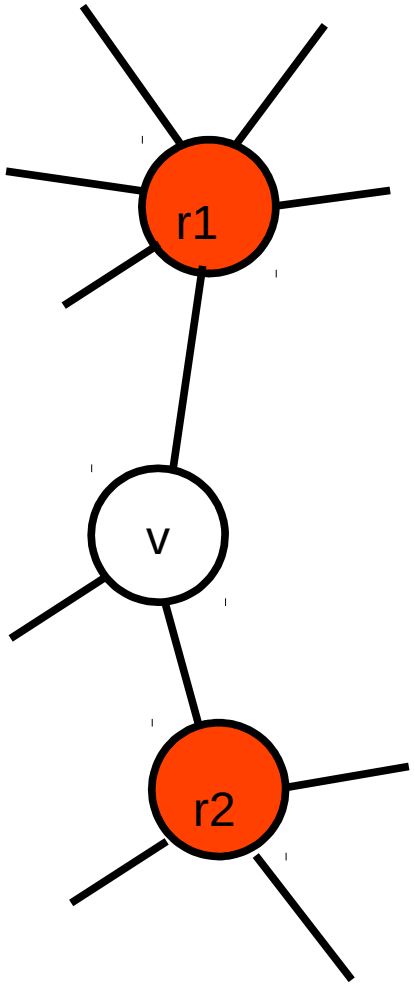
VISIBILITY: 2-HOP – BLOCK ALGORITHM

Theorem 3: Algorithm BLOCK fills the area represented by a connected graph in the ASYNC model by robots having a

- visibility range of 2 hops,
- $O(\log \Delta)$ bits of persistent storage, and using
- $\Delta + 4$ colors, including the color when the light is off.

MULTIPLE DOORS – K-DOOR-BLOCK ALGORITHM

- k Doors, k ≥2
- k chains
- Assume, robots entering from different doors have distinct colors.
- Priority protocol:
  - We define a strict total order between these colors, called priority order.
  - Taking leadership:
    new Leader takes the color of the old (Finished) Leader



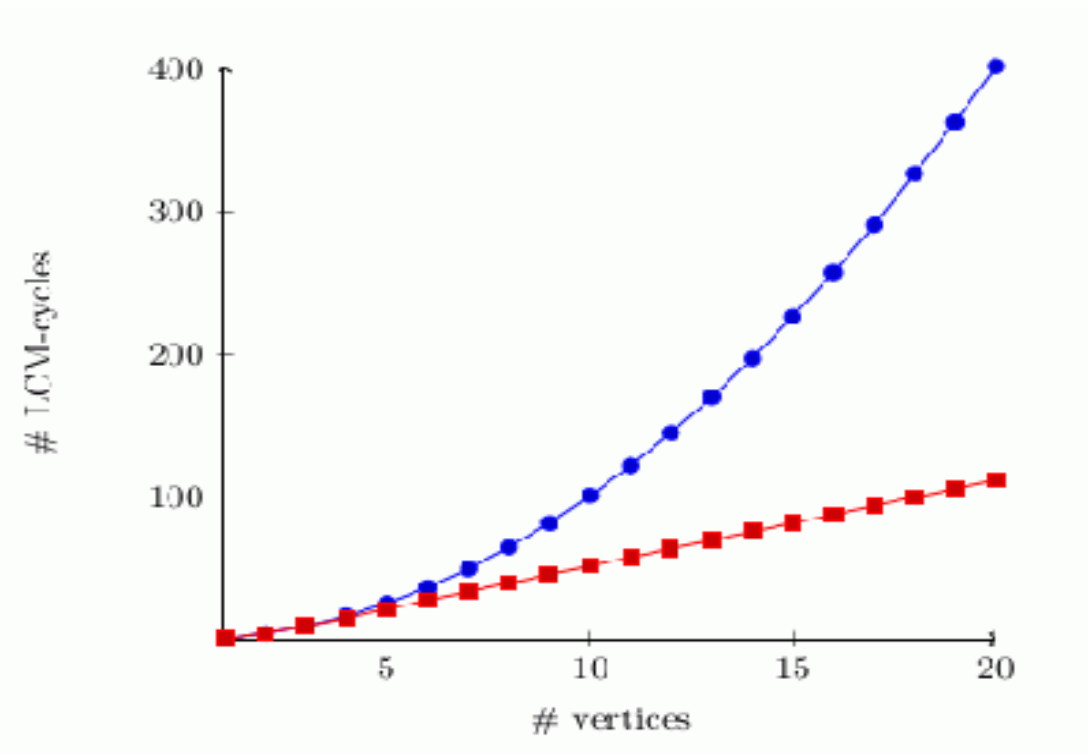
MULTIPLE DOORS – K-DOOR-BLOCK ALGORITHM

Theorem 4: Algorithm BLOCK extended with the Priority protocol solves the k-Door Filling problem, $k \geq 2$, in the ASYNC model with robots having a

- visibility range of 2 hops,
- $O(\log \Delta)$ bits of persistent memory and using
- $\Delta + k + 4$ colors including the color when the light is off.
- BLOCK needs $O(n)$ asynchronous rounds.

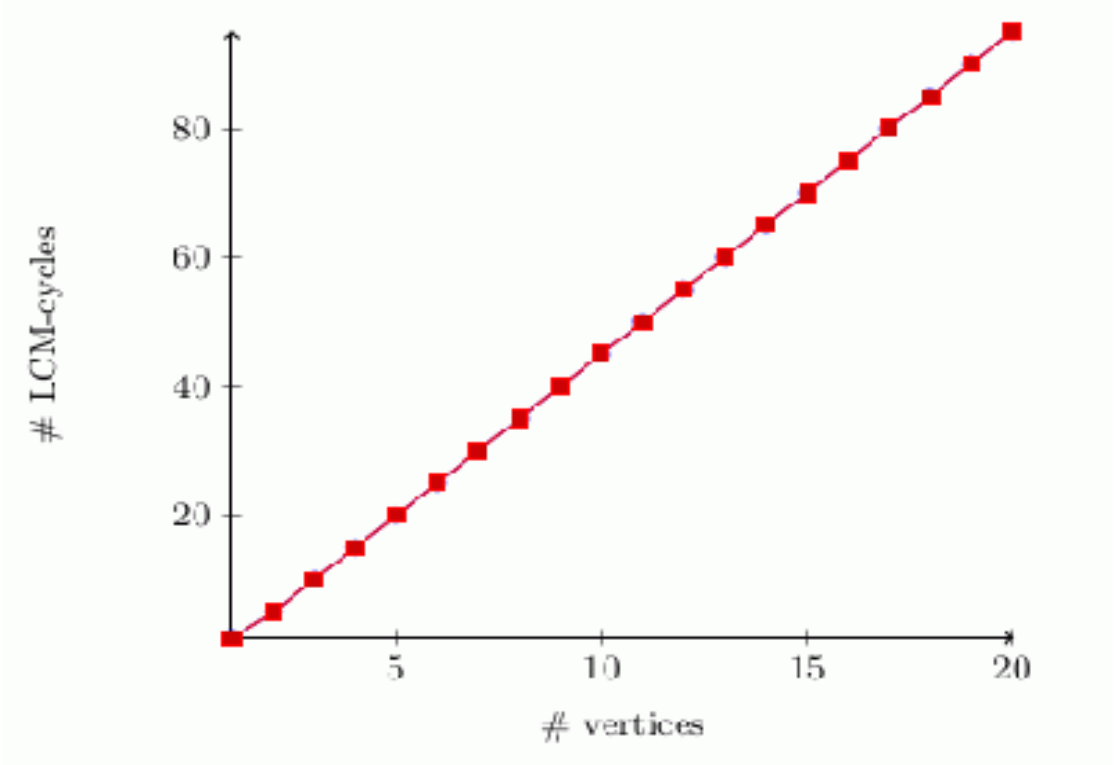
SIMULATIONS

Synchronous scheduler
Line graph, n = 1,…,20



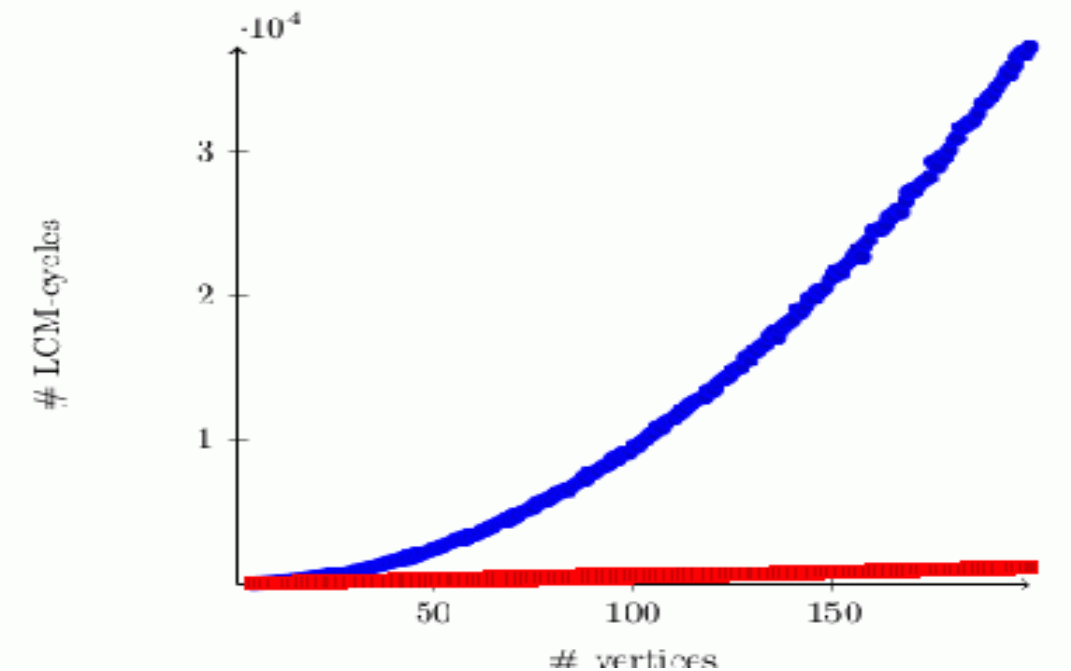
SIMULATIONS

Synchronous scheduler
Star graph, n = 1,…,20



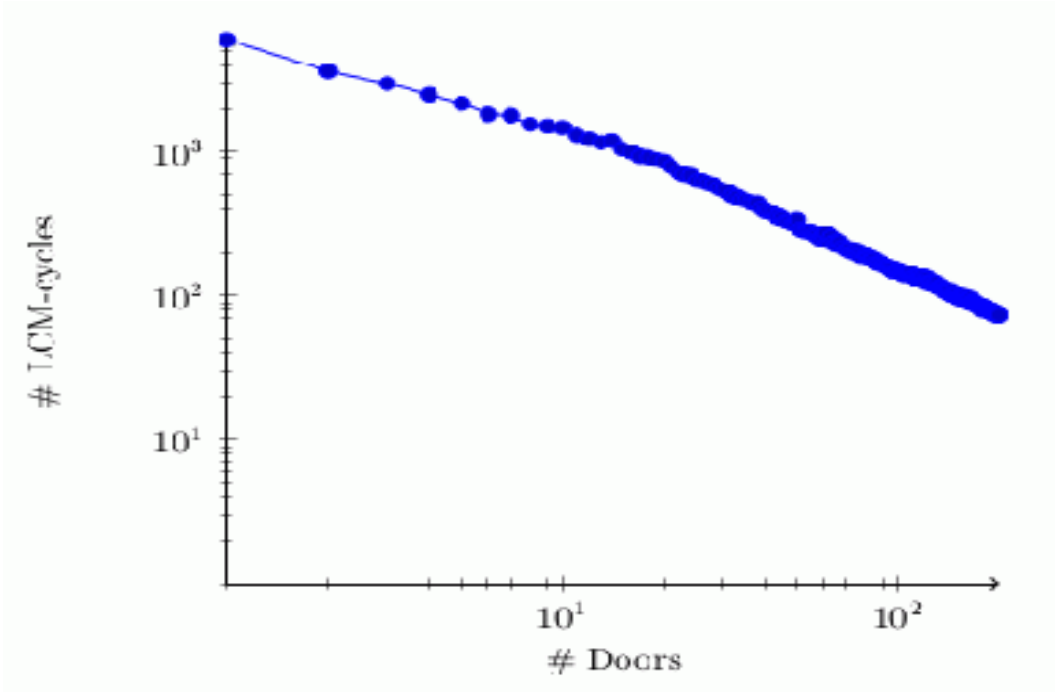
SIMULATIONS

Synchronous scheduler
Delaunay graph, vertices distributed uniformly at random in $[0,1]^2$
n = 3,…,200
50 runs for each n



SIMULATIONS

Synchronous scheduler
Delaunay graph, vertices distributed uniformly at random in $[0,1]^2$
n = 1000
k = 1,…,200
50 runs for each k



SUMMARY

Method	FSYNC/ ASYNC	Doors	Viewing range	Runtime #async rounds	Persistent memory bits	Colors	Area (Orthogonal/ Arbitrary)
PACK	ASYNC	Single	1	$O(n^2)$	$O(\log \Delta)$	$\Delta+4$	A
Mod-PACK	ASYNC	Single	1	$O(n^2 \log \Delta)$	$O(\log \Delta)$	$O(1)$	A
BLOCK	ASYNC	Single	2	$O(n)$	$O(\log \Delta)$	$\Delta+4$	A
k-Door-BLOCK	ASYNC	Multiple	2	$O(n)$	$O(\log(\Delta+k))$	$\Delta+k+4$	A

First asymptotic bounds for filling in the ASYNC model.
Only termination in finite time has been proven in previous works.

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Open question:

- Can the runtime be reduced for robots with visibility range 1?

Thank you for your attention!