Analysis of Distributed Systems

Exercises

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Theme I

Part I/a

Agenda

Exercise 1.1

*Create a Petri net illustrating a vending machine where first we have to choose between the two possible products (***little_choc*** and ***big_choc***) and then pay for it! Be the price of little_choc 10 and the price of big_choc 20 coins; and let accept the machine precisely 10 coins at a time!*

Exercise 1.2

Extend the previous net to illustrate a machine, which can accept 10 coins **or** *5 coins at a time!*

Exercise 1.3

Create a Petri net illustrating the following registration process! There are 200 admitted students, who can enroll to the University. The enrolled students can register to Course "A", but the maximum number of the registered students can not be more than 20.

Exercise 1.4

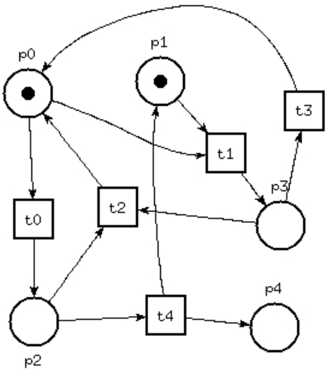
Extend the previous net to let the enrolled students to register to Course "B" too, where the maximum number is 23!

Exercise 1.5

Extend the previous net to let the registered students to deregister from any Course!

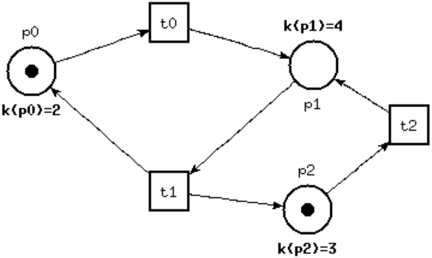
Exercise 1.6

Calculate the enabled firing sequences of the following Petri net!



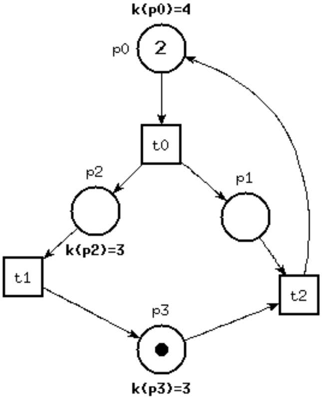
Exercise 1.7

Apply the complementary place transformation to the following Petri net!



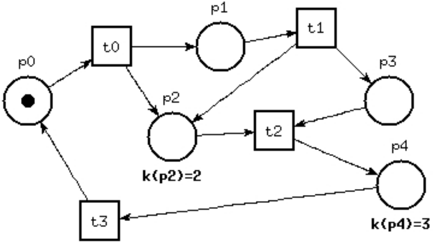
Exercise 1.8

Apply the complementary place transformation to the following Petri net!



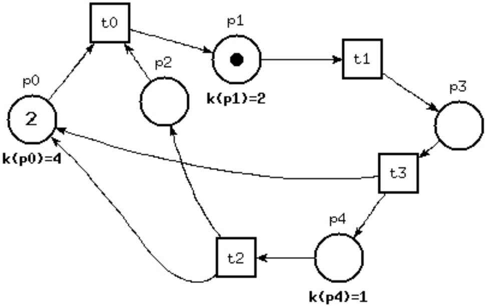
Exercise 1.9

Calculate the enabled firing sequences of the following Petri net! (Use the strict firing rule!)



Exercise 1.10

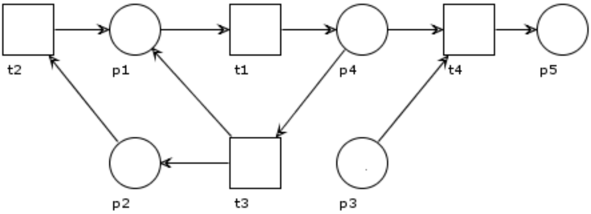
Calculate the enabled firing sequences of the following Petri net! (Use the strict firing rule!)



Agenda

Exercise 2.1

Are M_i and M_j markings reachable from the given M_0 markings? If yes, give the firing sequence too!



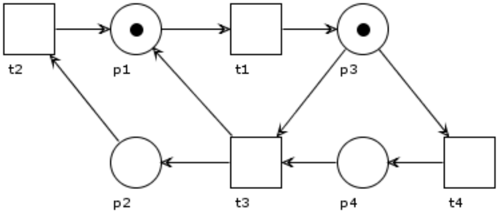
M_0

- A : $\{1, 0, 0, 0, 0\}$
- B : $\{1, 0, 1, 0, 0\}$

$M_i = \{2, 2, 0, 0, 0\}$
$M_j = \{2, 0, 0, 2, 1\}$

Exercise 2.2

Calculate the set of all possible markings reachable from M_0*!*



Exercise 2.3

Check the following properties of the previously shown (in exercise 2.1, 2.2) Petri nets!

- *Boundedness*
- *Safety*
- *Liveness*

Exercise 2.4

Modify the Petri net seen in Exercise 2.1. You can

- *add* **one** *extra edge*
- *use* **3** *tokens*

to create a $k = 4$ *bounded (and not* $k = 1, 2, 3$ *bounded) Petri net.*

Exercise 2.5

Find the smallest safe Petri net!

Exercise 2.6

Construct an L_0 live Petri net. Try to find the simplest one!

Exercise 2.7

Construct a strictly L_1 live Petri net. Try to find the simplest one!

Exercise 2.8

Construct a strictly L_2 *live Petri net. Try to find the simplest one!*

Exercise 2.9

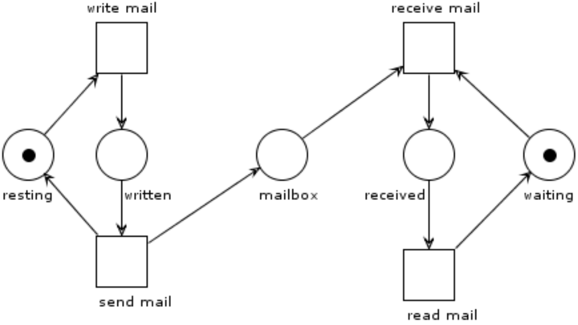
Construct a strictly L_3 live Petri net. Try to find the simplest one!

Exercise 2.10

Construct a strictly L_4 *live Petri net. Try to find the simplest, non-trivial one!*

Exercise 2.11

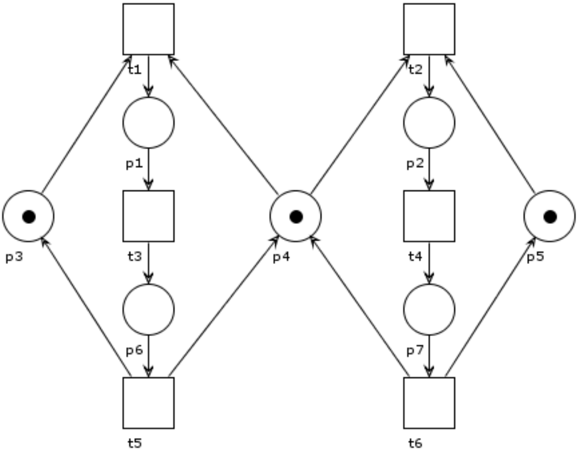
The following Petri net models an unbounded mailbox. Extend it so that it will have an upper bound on the mailbox capacity (#sent mails - #received mails). Try to minimize the modifications!



Agenda

Exercise 3.1

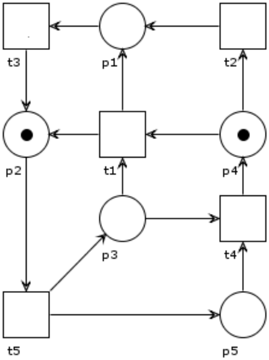
Is the following Petri net reversible? Has it other home state(s) except the initial marking? What could be modeled with this net?



Exercise 3.2

Are the following markings coverable in this Petri net with the given initial marking?

- M_1 : $\{0, 0, 0, 0, 0\}$
- M_2 : $\{0, 1, 0, 1, 0\}$
- M_3 : $\{0, 0, 1, 1, 1\}$
- M_4 : $\{0, 0, 1, 2, 0\}$
- M_5 : $\{1, 1, 1, 1, 1\}$



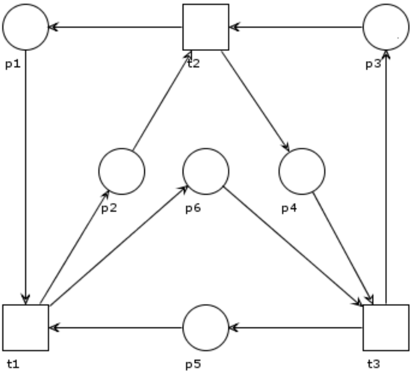
Exercise 3.3

Are the Petri nets of Exercise 3.1 and 3.2 persistent? Why? If not, how could you modify the nets to be persistent?

Exercise 3.4

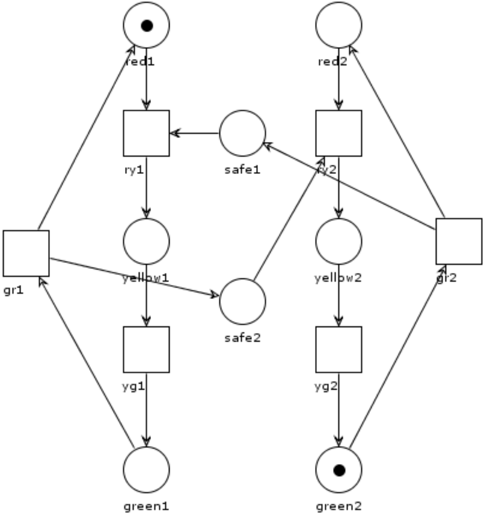
Calculate the following synchronic distances in case of each initial marking!

- $M_0 = (1, 0, 1, 0, 1, 0) : d_{1,2}, d_{3,2}$
- $M_1 = (1, 1, 0, 0, 1, 0) : d_{1,3}, d_{2,3}$
- $M_2 = (1, 1, 2, 0, 1, 1) : d_{2,3}$



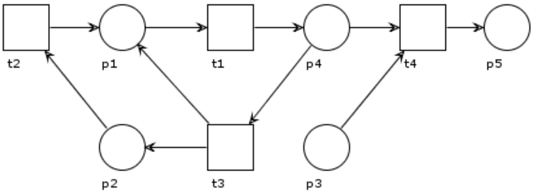
Exercise 3.5

This net models a safe and fair protocol for two traffic lights. Apply behavioral preserving reductions to shrink the net!



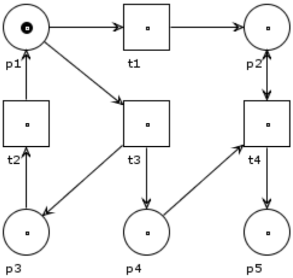
Exercise 3.6

Apply behavioral preserving reductions to shrink the net!



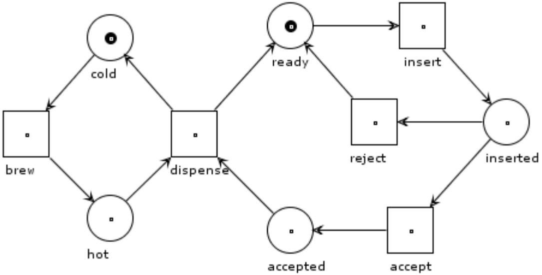
Exercise 3.7

Calculate the coverability tree/graph of the following Petri net!



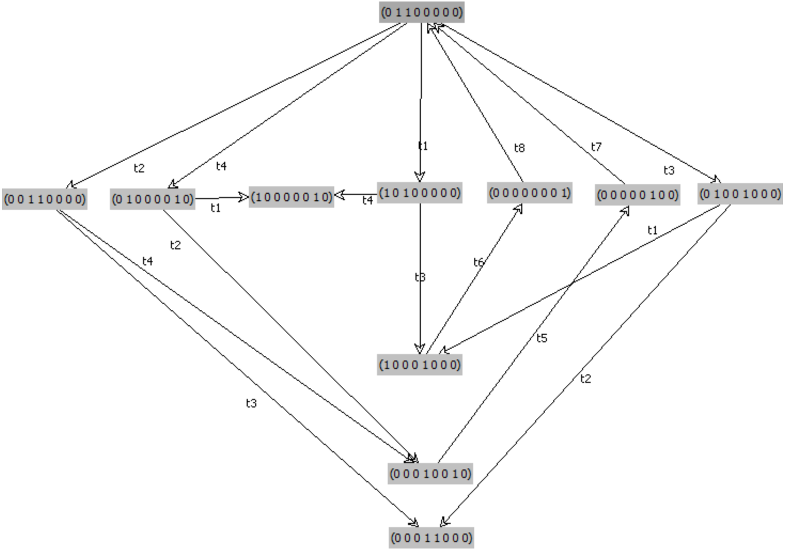
Exercise 3.8

Calculate the reachability tree/graph of the following Petri net!



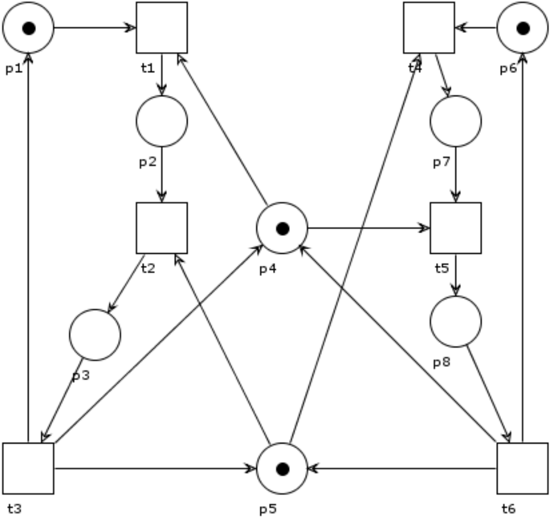
Exercise 3.9

Construct a safe Petri net, based on the following reachability graph!



Exercise 3.10

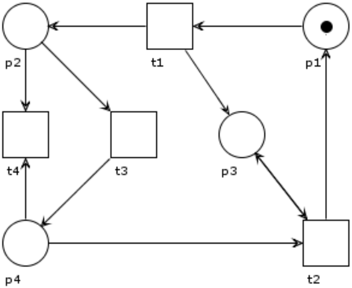
Is the following Petri net strongly connected?



Agenda

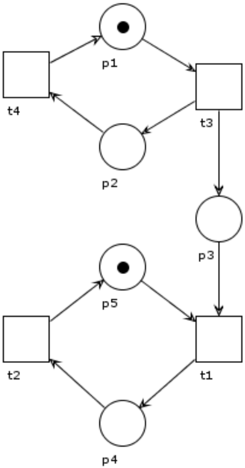
Exercise 4.1

Categorize the following Petri net. Also check its liveness and safeness properties!



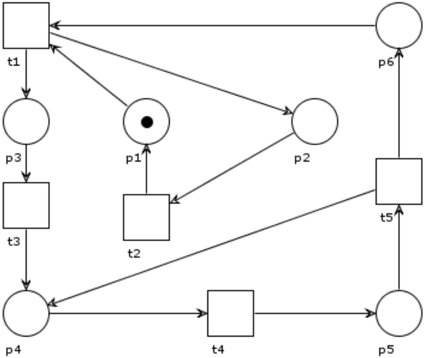
Exercise 4.2

Categorize the following Petri net. Also check its liveness and safeness properties!



Exercise 4.3

Categorize the following Petri net. Also check its liveness and safeness properties!



Exercise 4.4

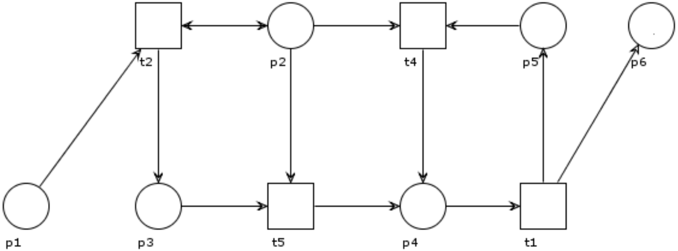
Modify the previously seen Petri net (in Exercise 4.3) to be EFC but not FC. Try to minimize the modifications!

Exercise 4.5

Modify the previously seen Petri net (in Exercise 4.3) to be AC but not EFC. Try to minimize the modifications!

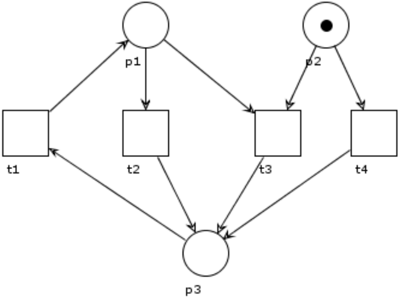
Exercise 4.6

Find source, sink places/transition, siphons and traps in the following Petri net!



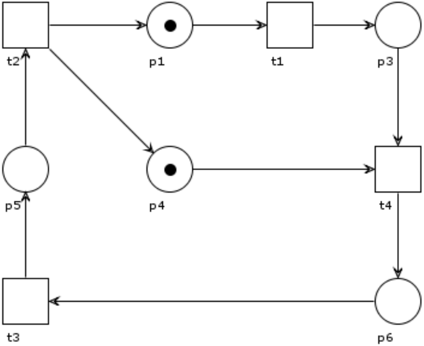
Exercise 4.7

Find as many Feedback Arc Set (FAS) as possible! Which is minimal/minimum?



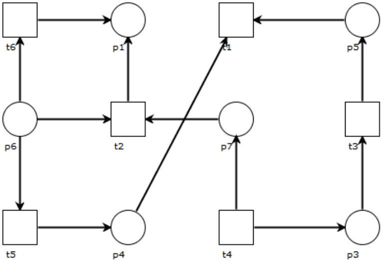
Exercise 4.8

Is the following strongly connected, live MG **safe***? Why (not)?*
Hint: think about FAS!



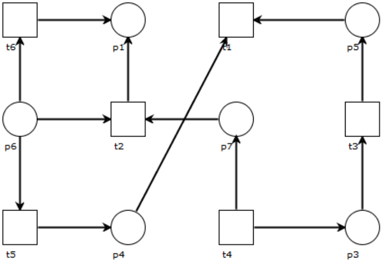
Exercise 4.9

Find an SM-component in the following Petri net!



Exercise 4.10

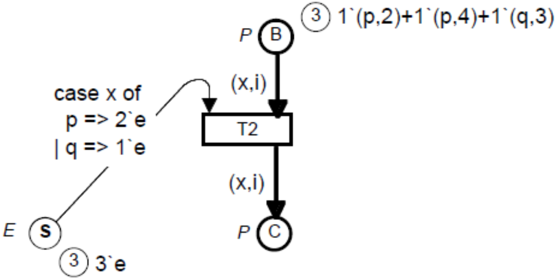
Find an MG-component in the following Petri net!



Agenda

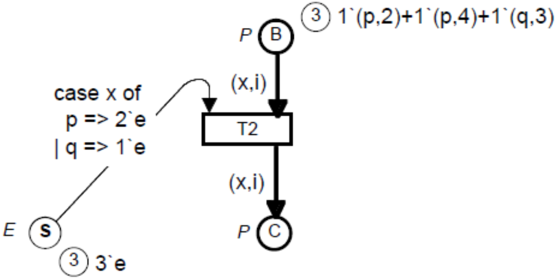
Exercise 5.1

Apply the $< x = p, i = 2 >$ *binding to the following CP net.*



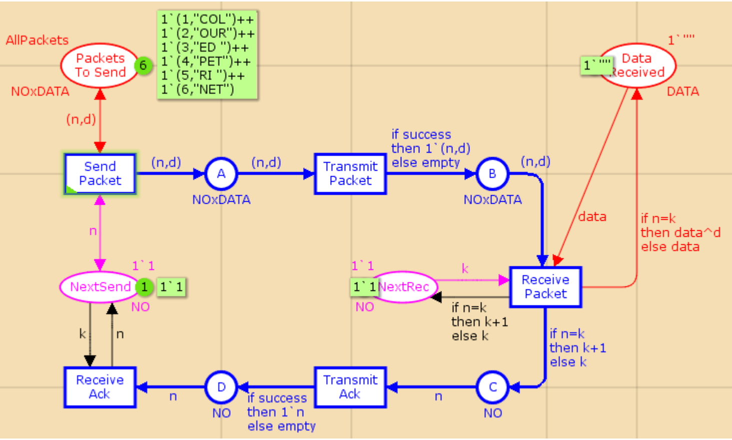
Exercise 5.2

Apply the $< x = q, i = 3 >$ binding to the following CP net.



Exercise 5.3

Consider the following simple protocol. Use the simulator named CPN Tools to perform some interactive and automatic simulations of the CPN model.



Exercise 5.4

Consider the previous simple protocol. When an acknowledgement is received by the sender, it updates the counter on NextSend according to the number contained in the acknowledgement. This implies that the counter on NextSend can be decreased when an "old" acknowledgement is received. Use the CPN simulator and interactive simulation to construct a scenario where this situation occurs.
Modify the CPN model such that the counter on NextSend will never be decreased. Use simulation to validate the modified model.

Exercise 5.5

Consider the previous simple protocol. For debugging reasons, extend the model with counters (LostPack, LostAck, Count) and bags(RecPack, RecAck)
Use simulation to validate the modified model.

Consider a system where a number of Chinese philosophers are situated around a circular table. In the middle of the table there is a delicious dish of rice, and between each pair of philosophers there is a single chopstick. Each philosopher alternates between thinking and eating. To eat, the philosopher needs two chopsticks, and he is only allowed to use the two which are situated next to him (on his left and right side). It is obvious that this restriction (lack of resources) prevents two neighbours from eating at the same time.

Exercise 5.6

Make a CPN model of the dining philosophers system with 5 philosophers. It is assumed that each philosopher simultaneously (and indivisibly) picks up his pair of chopsticks. Analogously, he puts them down in a single indivisible action. Make a simulation of the CPN model.

Exercise 5.7

Modify the CPN model, so that each philosopher takes the two chopsticks one at a time (but now in an arbitrary order, which may change from time to time). But, he puts down the two chopsticks simultaneously. Make a simulation of the new CPN model. Does the modification change the overall behaviour of the system (e.g., with respect to deadlocks and fairness between the philosophers)?

Exercise 5.8

We have two philosophers outside the room, each of them has one chopstick in his hand. When a philosopher enters the room, he puts down his chopstick and starts to think. When there is at least two unused chopsticks, a philosopher may start to eat by picking up two chopsticks. When he is finished, either puts down both chopsticks, or he gets food poisoning and rans out of the room, taking one chopstick with him.

A small model railway has a circular track with two trains a and b, which move in the same direction. The track is divided into seven different sectors $S = \{s_1, ...s_7\}$. At the start of each sector a signalpost indicates whether a train may proceed or not. To allow a train to enter a sector s_i it is required that this sector and also the next sector are empty.

Exercise 5.9

Describe the train system by a Petri-net. Each sector s_i may be represented by three places O_{ia} (sector s_i occupied by a), O_{ib} (sector s_i occupied by b) and E_i (sector s_i is empty).

Exercise 5.10

Describe the same system by a colored Petri-net where each sector is described by two places O_i (sector s_i is occupied) and E_i (sector s_i is empty).

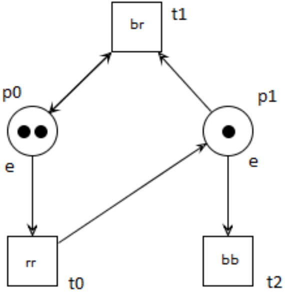
Theme II

Part I/b

Agenda

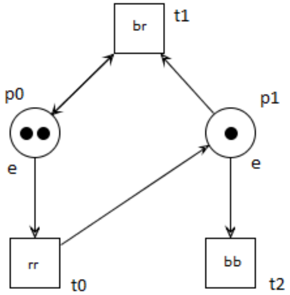
Exercise 1.1

Formalize the following labelled Petri net.



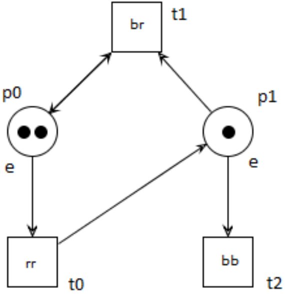
Exercise 1.2

Examine the behavioral properties (boundedness, safety) of the following Petri net.



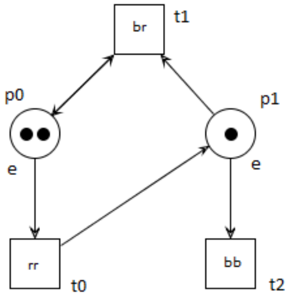
Exercise 1.3

Construct the coverability graph of the following Petri net.



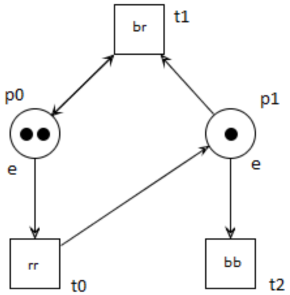
Exercise 1.4

Try to apply the following step sequence: $\{ \{rr\}, \{br\}, \{rr\}, \{rr\}, \{bb\} \}$
What will be the outcome?



Exercise 1.5

Try to apply the following step sequence: $\{ \{rr, br\}, \{rr, bb\}, \{bb\} \}$
What will be the outcome?



Exercise 1.6

Draw a Petri net based on the following:

$\Sigma_0 = (S_0, T_0, W_0, \lambda_0, M_0)$
$S_0 = \{p_0, p_1, p_2, p_3, p_4\}$
$T_0 = \{t_0, t_1, t_2, t_3\}$
$W_0 = ((TS \cup ST) \times \{1\}) \cup (((S \times T) \setminus ST \cup (T \times S) \setminus TS) \times \{0\})$
$\lambda_0 = \{(p_0, i), (p_1, i), (p_2, i), (p_3, i), (p_4, i), (t_0, write), (t_1, send),$
$(t_2, receive), (t_3, read)\}$
$M_0 = \{(p_0, 1), (p_4, 1)\}$

where

$TS = \{(t_0, p_1), (t_1, p_0), (t_1, p_2), (t_2, p_3), (t_3, p_4)\}$ and
$ST = \{(p_0, t_0), (p_1, t_1), (p_2, t_2), (p_3, t_3), (p_4, t_2)\}$

Exercise 1.7

Examine the behavioral properties (boundedness, safety) of the previous Petri net.

Exercise 1.8

Construct the coverability graph of the previous Petri net.

Exercise 1.9

Try to apply the following step sequence on the previous Petri net: $\{\{write\}, \{send\}, \{write\}, \{receive\}, \{send\}, \{read\}, \{receive\}, \{write\}\}$
What will be the outcome?

Exercise 1.10

Try to apply the following step sequence on the previous Petri net: $\{\{write\}, \{send\}, \{write, receive\}, \{send\}, \{write, read, receive\}\}$
What will be the outcome?

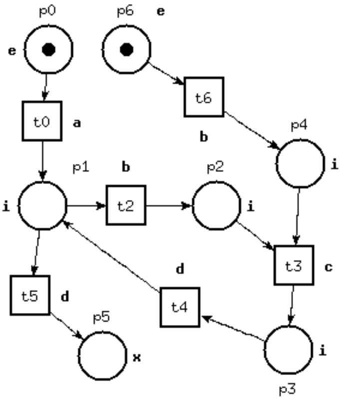
Agenda

Exercise 2.1

Please create a T-restricted, ex-restricted and ex-directed labelled Petri net.

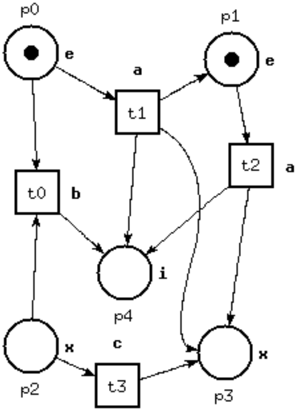
Exercise 2.2

Consider the following labelled Petri net. Is the net ex-directed and ex-exclusive? Please give the transitions which are independent of transition t_0.



Exercise 2.3

Consider the following labelled Petri net. Please give the ex-asymmetric transitions of the net.



Exercise 2.4

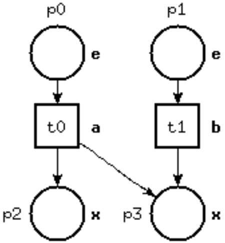
Please create a dinamic box.

Exercise 2.5

Please create a static box which is not ex-exclusive.

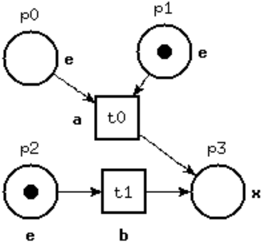
Exercise 2.6

Consider the following labelled Petri net. Is the net a static box?



Exercise 2.7

Consider the following labelled Petri net. Is the net a dinamic box?

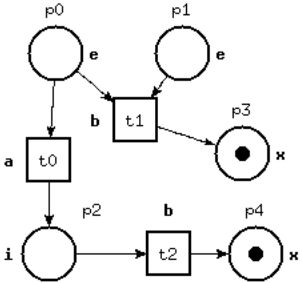


Exercise 2.8

Is the previous net ex-exclusive?

Exercise 2.9

Consider the following labelled Petri net. Is the net an exit box?



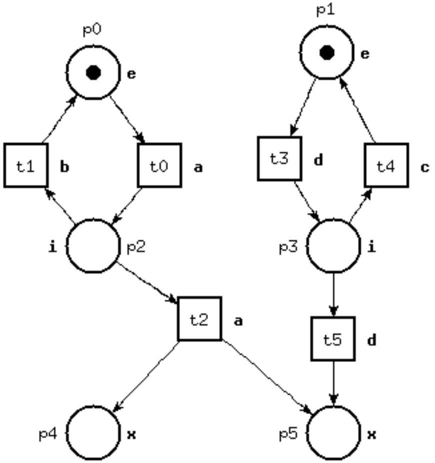
Exercise 2.10

Which transitions are independent in the previous net? Is the net ex-exclusive?

Agenda

Exercise 3.1

Consider the following Petri box. Is the step sequence $\rho = (\{t_0, t_3\});\ (\{t_4\} : \{t_1\}^+);\ (\{t_3\}, \{t_1\}^-)$ *enabled from the initial complex marking* $\mathcal{M} = ((1, 1, 0, 0, 0, 0), \emptyset)$*?*



Exercise 3.2

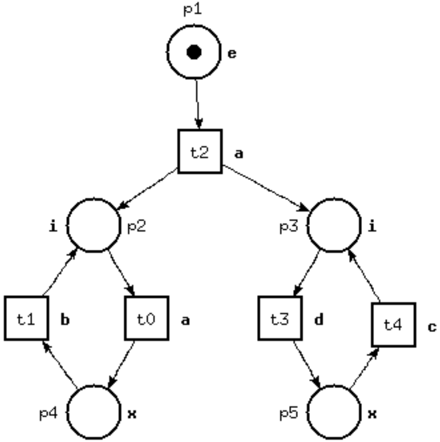
Consider the Petri box in Example 3.1. Is the complex marking $\mathcal{M}' = ((0,0,0,0,1,1),\{t_5\})$ *reachable from the initial marking?*

Exercise 3.3

Consider the Petri box in Example 3.1. Calculate the complex markings directly reachable from marking $\mathcal{M}'' = ((1,0,0,0,0,0),\{t_5\})$.

Exercise 3.4

Consider the following Petri box. Is the step sequence $\rho = (\{t_2\});\ (\{t_3\} : \{t_0\}^+);\ (\{t_1, t_4\}^+)$ *enabled from the initial complex marking* $\mathcal{M} = ((1,0,0,0,0), \emptyset)$*?*

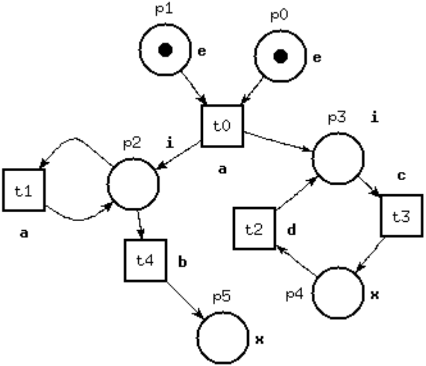


Exercise 3.5

Consider the Petri box in Example 3.4. Is the step sequence $\rho = (\{t_2\});\ (\{t_0\} : \{t_3\}^+);\ (\{t_1\}^+ : \{t_3\}^-)$ *enabled from the initial marking?*

Exercise 3.6

Consider the following Petri box. Is the step sequence $\rho = (\{t_0\});\ (\{t_3\} : \{t_1\}^+);\ (\{t_2\}^+ : \{t_1\}^-)$ *enabled from the initial complex marking* $\mathcal{M} = ((1,1,0,0,0,0), \emptyset)$*?*

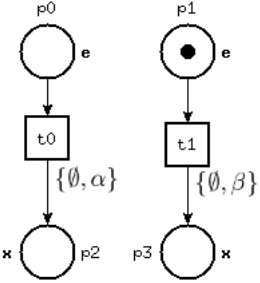


Exercise 3.7

Consider the Petri box in Example 3.6. Is the complex marking $\mathcal{M}' = ((0,0,0,0,0,0), \{t_1, t_2\})$ *reachable from the initial marking?*

Exercise 3.8

Consider the following Petri box (Σ) *and the transformational relabelling* $\rho = \big\{(\{\alpha\}, \gamma), (\{\alpha, \alpha\}, \alpha), (\{\alpha, \beta\}, \beta)\big\}$*. Calculate the interface change of* Σ *according to relabelling* ρ*.*

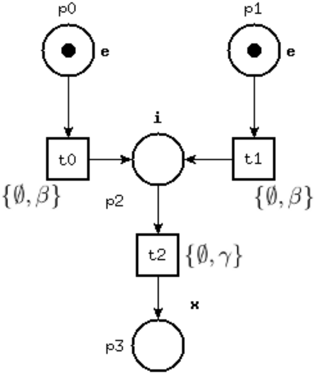


Exercise 3.9

Consider the Petri box in Example 3.8 and the transformational relabelling $\rho = \{(\{\beta\}, \alpha), (\{\alpha\}, \beta)\}$*. Calculate the interface change of* Σ *according to relabelling* ρ*.*

Exercise 3.10

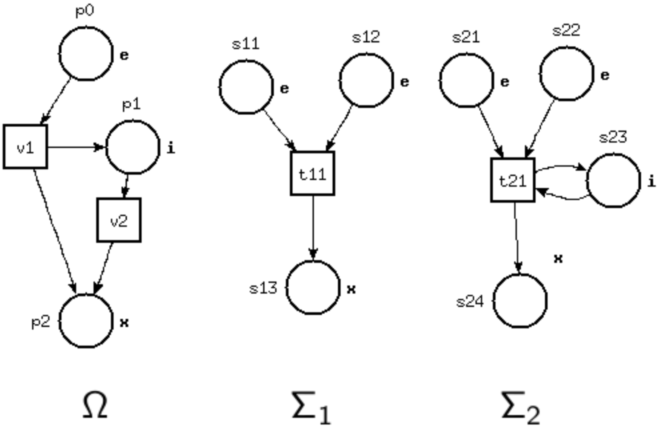
Consider the following Petri box (Σ) *and the transformational relabelling* $\rho = \{(\{\alpha\}, \gamma), (\{\gamma, \beta\}, \alpha), (\{\beta, \beta\}, \beta)\}$*. Calculate the interface change of* Σ *according to relabelling* ρ*.*



Agenda

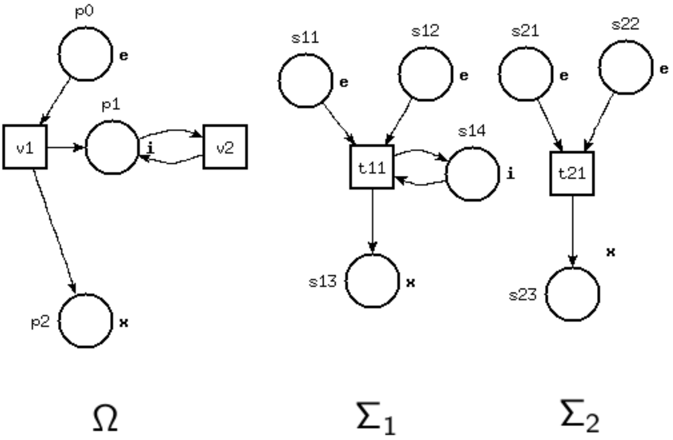
Exercise 4.1

Consider the following operator box Ω *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2\}$. *Calculate the transition refinement of* Σ *according to* Ω.



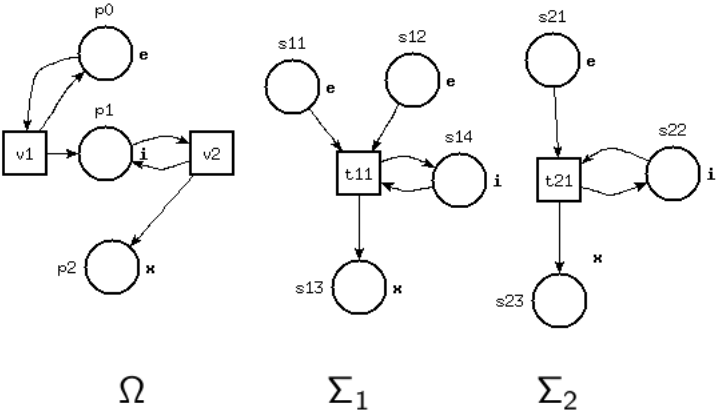
Exercise 4.2

Consider the following operator box Ω *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2\}$. *Calculate the transition refinement of* Σ *according to* Ω.



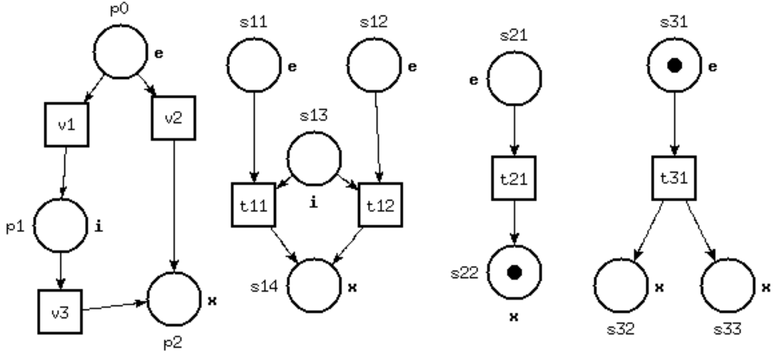
Exercise 4.3

Consider the following operator box Ω *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2\}$. *Calculate the transition refinement of* Σ *according to* Ω.



Exercise 4.4

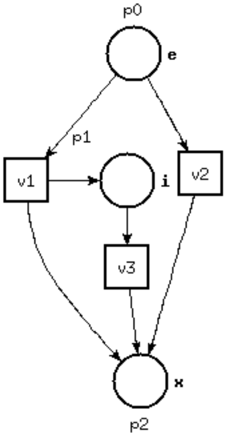
Consider the following operator box Ω *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2, \Sigma_3\}$. *Calculate the transition refinement of* Σ *according to* Ω.



Ω Σ_1 Σ_2 Σ_3

Exercise 4.5

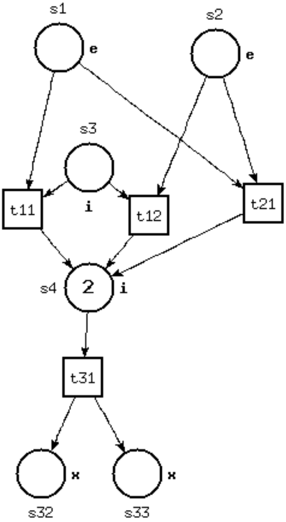
Consider the Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2, \Sigma_3\}$ *in Example 4.4 and the following operator box* Ω*. Calculate the transition refinement of* Σ *according to* Ω*.*



Ω

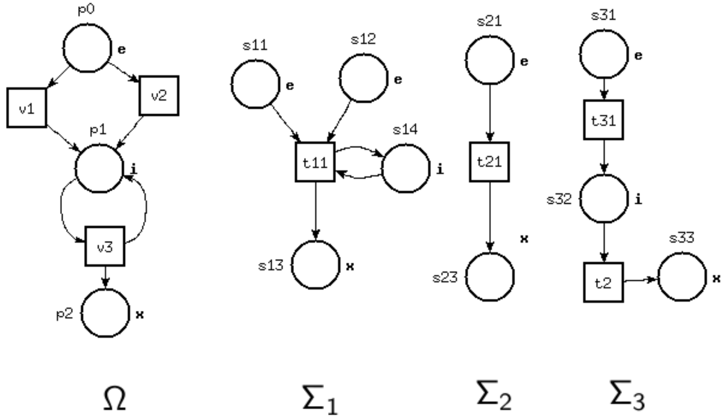
Exercise 4.6

Consider the Ω-tuple $\Sigma = \{\Sigma_1, \Sigma_2, \Sigma_3\}$ in Example 4.4. Can you give an operator box Ω according to which the transition refinement of Σ will be the following?



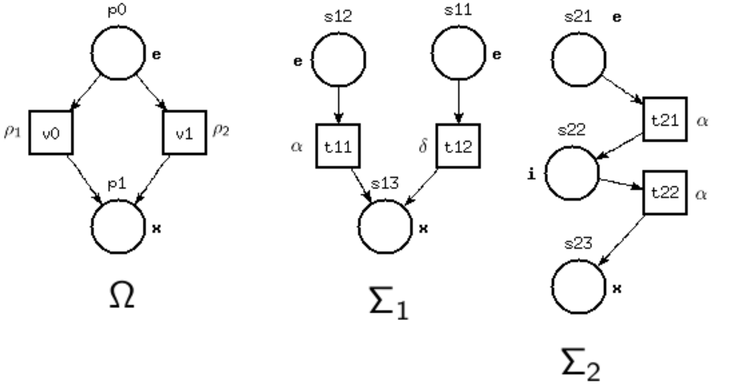
Exercise 4.7

Consider the following operator box Ω *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2, \Sigma_3\}$. *Calculate the transition refinement of* Σ *according to* Ω.



Exercise 4.8

Consider the following operator box Ω*, the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2\}$ *and the relabellings* $\rho_1 = \{(\{\alpha, \delta\}, \beta)\}$*,* $\rho_2 = \{(\{\alpha, \alpha\}, \delta)\}$.
Calculate the net refinement of Σ *according to* Ω*.*

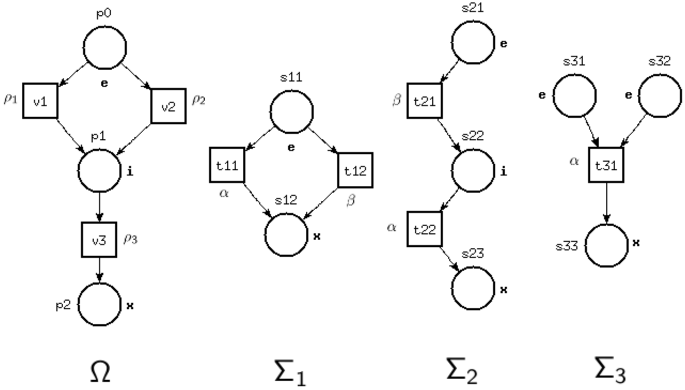


Exercise 4.9

Consider the Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2\}$ *and the operator box* Ω *from the Example 4.8 with relabellings*
$\rho_1 = \big\{(\{\alpha\}, \alpha), (\{\alpha, \delta\}, \delta)\big\}$, $\rho_2 = \big\{(\{\alpha\}, \delta)\big\}$.
Calculate the net refinement of Σ *according to* Ω.

Exercise 4.10

Consider the following operator box Ω *with relabellings* $\rho_1 = \{(\{\alpha\}, \alpha), (\{\alpha, \beta\}, \beta)\}$, $\rho_2 = \{(\{\alpha, \beta\}, \delta)\}$, $\rho_3 = \{(\{\alpha\}, \delta)\}$ *and the* Ω*-tuple* $\Sigma = \{\Sigma_1, \Sigma_2, \Sigma_3\}$. *Calculate the net refinement of* Σ *according to* Ω.



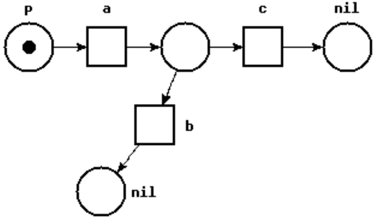
Theme III

Part II

Agenda

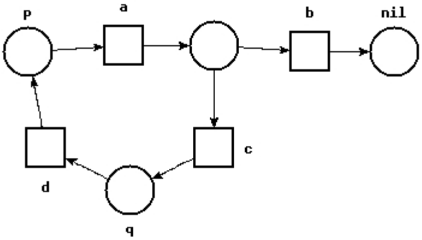
Exercise 1.1

Create an LTS, which simulates the same process as the following Petri net!



Exercise 1.2

Create an LTS, which simulates the same process as the following Petri net!



Exercise 1.3

Does the process $p = a(bnil + c(dnil + bdnil))$ *correspond to the environment* $e = acbnil$ *or not?*

Exercise 1.4

Does the process $p = abdnil + a(dnil + cnil)$ *correspond to the environment* $e = a(bnil + cnil)$ *or not?*

Exercise 1.5

Does the process $p = abnil + adnil$ *correspond to the environment* $e = abnil + adnil$ *or not?*

Exercise 1.6

Calculate 5 environments to which the process $p = abnil + adnil$ *corresponds!*

Exercise 1.7

Calculate the function $\tau(a(bnil + c(dnil + bdnil))$*!*

Exercise 1.8

Calculate the function $\tau'(a(bnil + c(dnil + bdnil))$*!*

Exercise 1.9

Prove the following equivalence
$\big(abnil + a(cnil + dnil)\big)\ equ_a\ \big(a(bnil + cnil) + adnil\big)$*!*

Exercise 1.10

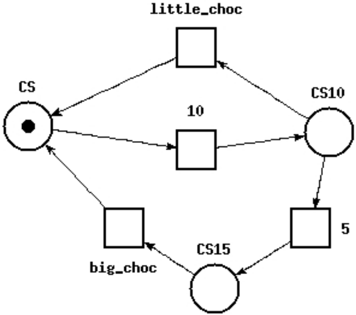
Prove the following equivalence

$$\big(a((bcnil + dnil) + d(cnil + nil))\big)\quad equ_a\ \big(ab(cnil + nil) + a(d(nil + cnil))\big)!$$

Agenda

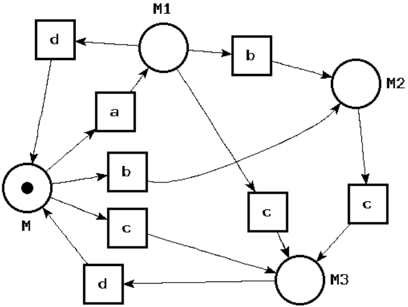
Exercise 2.1

Create a CSP, which simulates the same process as the following Petri net!



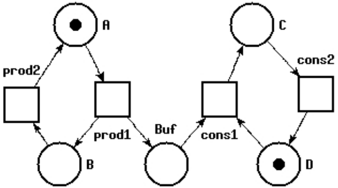
Exercise 2.2

Create a CSP, which simulates the same process as the following Petri net!



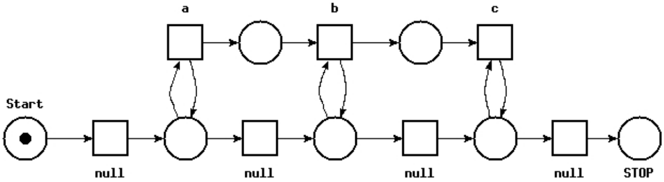
Exercise 2.3

Create a CSP, which simulates the same process as the following Petri net!



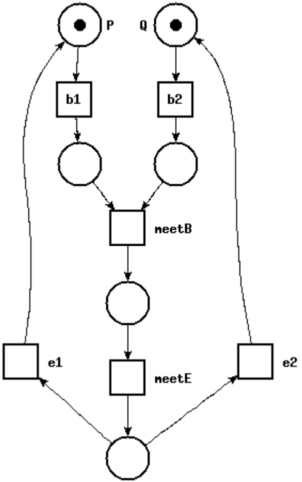
Exercise 2.4

Create a CSP, which simulates the same process as the following Petri net!



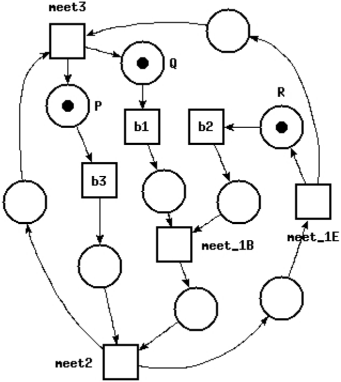
Exercise 2.5

Create a CSP, which simulates the same process as the following Petri net!



Exercise 2.6

Create a CSP, which simulates the same process as the following Petri net!



Exercise 2.7

Create a CSP, which simulates the same as the following LTS process!

$a(bnil + cnil) + bdnil$

Exercise 2.8

Create a CSP, which simulates the same as the following LTS process!

$(a(bnil + cnil) + dnil) + b(cnil + cdnil)$

Exercise 2.9

Create a Petri net, which simulates the same as the following CSP process!
$P \mid\mid Q$, *where*
$P = a \to b \to d \to P$ *and* $\alpha P = \{a, b, d\}$,
$Q = b \to c \to Q$ *and* $\alpha Q = \{b, c\}$.

Exercise 2.10

Create an LTS process, which simulates the same as the following CSP process!
$P \mid\mid Q$, *where*
$P = a \to b \to d \to STOP$ *and* $\alpha P = \{a, b, d\}$,
$Q = c \to b \to STOP$ *and* $\alpha Q = \{b, c\}$.

Agenda

Exercise 3.1

Let be $P = a \rightarrow b \rightarrow P$, $\alpha P = \{a, b\}$.
Let be $Q = c \rightarrow b \rightarrow Q$, $\alpha Q = \{b, c\}$.
Calculate $P \mid\mid Q$ *!*

Exercise 3.2

Let be $P = b \rightarrow a \rightarrow P$, $\alpha P = \{a, b\}$.
Let be $Q = b \rightarrow c \rightarrow Q$, $\alpha Q = \{b, c\}$.
Calculate $P \mid\mid Q$ *!*

Exercise 3.3

Let be $P = a \rightarrow b \rightarrow P$, $\alpha P = \{a, b\}$.
Let be $Q = b \rightarrow c \rightarrow Q$, $\alpha Q = \{b, c\}$. *Calculate* $P \mid\mid Q$ *!*

Exercise 3.4

Let be $P = (a \rightarrow b \rightarrow P \mid b \rightarrow c \rightarrow P)$, $\alpha P = \{a, b, c\}$.
Let be $Q = a \rightarrow (c \rightarrow Q \mid b \rightarrow Q)$, $\alpha Q = \{a, b, c\}$.
Calculate $P \mid\mid Q$ *!*

Exercise 3.5

Let be $P = x \rightarrow y \rightarrow z \rightarrow y \rightarrow P$, $\alpha P = \{x, y, z\}$.
Let be $Q = y \rightarrow w \rightarrow y \rightarrow Q$, $\alpha Q = \{y, w\}$.
Calculate $P \mid\mid Q$ *!*

Exercise 3.6

Let be $P = (a \rightarrow b \rightarrow P \mid b \rightarrow c \rightarrow P)$, $\alpha P = \{a, b, c\}$.
Let be $Q = a \rightarrow (c \rightarrow Q \sqcap b \rightarrow Q)$, $\alpha Q = \{a, b, c\}$.
Calculate $P \mid\mid Q$ *!*

Exercise 3.7

Let be $P = (a \rightarrow b \rightarrow P \mid d \rightarrow b \rightarrow c \rightarrow P)$,
$\alpha P = \{a, b, c, d\}$.
Calculate $P \setminus \{b\}$ *!*

Exercise 3.8

Let be $P = (a \to b \to c \to P) \,\square\, (a \to c \to d \to P)$,
$\alpha P = \{a, b, c, d\}$,
$Q = a \to \big(b \to (c \to Q \,\square\, b \to Q) \mid c \to (d \to Q \,\square\, a \to Q)\big)$,
$\alpha Q = \{a, b, c, d\}$
Calculate $P \,||\, Q$ *!*

Exercise 3.9

Let be $P = a \to b \to c \to d \to P$, $\alpha P = \{a, b, c, d\}$.
Let be $Q = e \to a \to e \to c \to Q$, $\alpha Q = \{a, c, e\}$.
Calculate $P \,||\, Q$ *!*

Exercise 3.10

Let be
$P = \big(x \to y \to (z \to P \,\square\, w \to P)\big) \,\square\, \big(x \to z \to w \to P\big)$,
$\alpha P = \{x, y, z, w\}$,
$Q = x \to \big(y \to (z \to Q \,\square\, y \to Q) \mid z \to (w \to Q \,\square\, x \to Q)\big)$,
$\alpha Q = \{x, y, z, w\}$
Calculate $P \,||\, Q$ *!*

Agenda

Exercise 4.1

Let be $R(a) = b \to c \to STOP$,
$R(b) = d \to STOP$, *and*
$W = x : \{a, b\} \to R(x)$.
Calculate $traces(W)$*!*

Exercise 4.2

Let be $P = b \to c \to STOP$,
$Q = d \to c \to STOP$.
Calculate $traces(P \sqcap Q)$*!*

Exercise 4.3

Let be $R = d \to c \to STOP$,
$W = b \to d \to c \to STOP$.
Calculate $traces(R \Box W)$*.!*

Exercise 4.4

Use the notations of the previous two exercises. Suppose
$\alpha(P \sqcap Q) = \alpha(R \Box W)$
Calculate $traces((P \sqcap Q) \mid\mid (R \Box W))$*!*

Exercise 4.5

Let be $VMS = coin \rightarrow choc \rightarrow VMS$.
Prove, that $traces(VMS) = \bigcup_{n \geq 0} \{s \mid s \ \leq \ < coin, choc >^n\}$ *!*

Exercise 4.6

Let be $P = a \rightarrow (b \rightarrow STOP \mid c \rightarrow STOP)$.
Prove, that P *sat* $((tr \downarrow a) \geq (tr \downarrow c))$ *!*

Exercise 4.7

Let be $P = b \rightarrow c \rightarrow STOP$, $\alpha P = \{b, c\}$,
$Q = a \rightarrow b \rightarrow STOP$, $\alpha Q = \{a, b\}$.
Prove, that $(P \mid\mid Q)$ *sat* $((tr \downarrow a) \leq (tr \downarrow b))$*!*

Exercise 4.8

Let be $P = b \rightarrow c \rightarrow STOP$, $\alpha P = \{b, c\}$,
$Q = a \rightarrow b \rightarrow STOP$, $\alpha Q = \{a, b\}$.
Prove, that $(P \ \sqcap \ Q)$ *sat* $((tr \downarrow a) + (tr \downarrow c) \leq 1)$*!*

Exercise 4.9

Let be $P = a \to (b \to c \to P \mid c \to d \to P)$.
Prove, that P *sat* $((tr \downarrow a) \leq 1 + (tr \downarrow b) + (tr \downarrow d))$ *!*

Exercise 4.10

Let be $P = a \to b \to P$, $(\alpha P = \{a, b\})$, *and*
$Q = b \to c \to Q$, $(\alpha Q = \{b, c\})$.
Prove, that $P || Q$ *sat* $(0 \ \leq \ ((tr \downarrow a) - (tr \downarrow c)) \ \leq \ 2)$*!*

Agenda

Exercise 5.1

$P = c1?x \rightarrow c2!(x + 5) \rightarrow STOP$

$Q = c2?x \rightarrow c3!(2 * x) \rightarrow STOP$

$P || Q = ?$

Exercise 5.2

$P = c1?x \rightarrow c2!(x + 5) \rightarrow STOP$

$Q = c2?x \rightarrow c3!(2 * x) \rightarrow STOP$

$(P || Q) \setminus \{c2\} = ?$

Exercise 5.3

$P = in?x \to c1!x \to c2?y \to out!y \to P$

$Q_i = c1?x \to c2!(x * i) \to Q_i$, *where* $i \in \mathbb{Z}$

$P || Q_{10} = ?$

Exercise 5.4

$P = in?x \to c1!x \to c2?y \to out!y \to P$

$Q_i = c1?x \to c2!(x * i) \to Q_i$, *where* $i \in \mathbb{Z}$

$(P || Q_5) \setminus \{c1, c2\} = ?$

Exercise 5.5

$P = in1?x \rightarrow c1!x \rightarrow c2?y \rightarrow out!y \rightarrow P$

$Q = in2?x \rightarrow c1?y \rightarrow c2!(x + y) \rightarrow Q$

$P||Q =?$

Exercise 5.6

$P = in1?x \rightarrow c1!x \rightarrow out!x \rightarrow c2?y \rightarrow out!y \rightarrow P$

$Q = c1?x \rightarrow in2?y \rightarrow c2!(x * y) \rightarrow Q$

$P||Q =?$

Exercise 5.7

$$P = (in?x \rightarrow out!(x, x+10) \rightarrow P)$$
$$(P \gg Q) \ sat \ \left(out \overset{2}{\leq} fv^*(in)\right)$$
$$, where \ fv(z) = z^2 + 10z + 5$$
$$Q = ?$$

Exercise 5.8

$$P = (in?x \rightarrow out!(x, 2*x+5) \rightarrow P)$$
$$R = (in?z \rightarrow out!(z_1 * z_2 + 7) \rightarrow R)$$
$$((P \gg Q) \gg R) \ sat \ \left(out \leq fv^*(in)\right)$$
$$, where \ fv(w) = 2w^3 + 5w^2 + 3w + 7$$
$$Q = ?$$

Exercise 5.9

$$P = (in?x \rightarrow out!(x,1,1) \rightarrow P)$$
$$R = (in?z \rightarrow out!z_2 \rightarrow R)$$
$$(P \gg Q \gg Q \gg Q \gg Q \gg R) \; sat \; \left(out \leq fv^*(in)\right)$$
$$,where \; fv(w) = \sum_{i=0}^{4} w^i$$
$$Q = ?$$

Exercise 5.10

$$P = (in?x \rightarrow out!(x,1,1) \rightarrow P)$$
$$R = (in?z \rightarrow out!z_2 \rightarrow R)$$
$$(P \gg Q_1 \gg Q_2 \gg Q_3 \gg Q_4 \gg R) \; sat \; \left(out \leq fv^*(in)\right)$$
$$,where \; fv(w) = \sum_{i=0}^{4} \frac{w^i}{i!}$$
$$Q_i = ?$$