Relational Design Theory

- Assess the quality of a schema
  - redundancy
  - integrity constraints
  - **Quality seal: normal forms (1-4, BCNF)**

- Improve the quality of a schema
  - **synthesis algorithm**
  - **decomposition algorithm**

- Construct a (high-quality) schema
  - start with universal relation
  - apply synthesis or decomposition algorithms

What is wrong with redundancy?

- Waste of storage space
  - importance is diminishing as storage gets cheaper
  - (disk density will even increase in the future)

- Additional work to keep multiple copies of data consistent
  - multiple updates in order to accomodate one event

- Additional code to keep multiple copies of data consistent
  - Somebody needs to implement the logic

Bad Schemas

ProfLecture						
PersNr	Name	Level	Room	Nr	Title	CP
2125	Sokrates	FP	226	5041	Ethik	4
2125	Sokrates	FP	226	5049	Mäeutik	2
2125	Sokrates	FP	226	4052	Logik	4
...	...	...	...	...	...	...
2132	Popper	AP	52	5259	Der Wiener Kreis	2
2137	Kant	FP	7	4630	Die 3 Kritiken	4

- Update-Anomaly
  - What happens when Sokrates moves to a different room?
- Insert-Anomaly
  - What happens if Roscoe is elected as a new professor?
- Delete-Anomaly
  - What happens if Popper does not teach this semester?

Multi-version Databases

- Storage becomes cheaper -> never throw anything away
  - It is more expensive to think about what to keep than simply to keep everything.

- Consequence 1: No delete
  - Instead, set a status flag to „deleted“
  - No delete anomalies (only wasted storage)
- Consequence 2: No update in place
  - Instead, create a new version of the tuple
  - No update anomalies (only wasted storage)
- Insert anomalies still exist, but not a big problem
  - Result in multiple NULL values, but no inconsistencies

- **NoSQL Movement: Denormalized data (XML is great!)**

Functional Dependencies

- Schema: $\mathcal{R} = \{A{:}D_A,\ B{:}D_B,\ C{:}D_C,\ D{:}D_D\}$
- Instance: R

- Let $\alpha \subseteq \mathcal{R},\ \ \beta \subseteq \mathcal{R}$
- $\alpha \rightarrow \beta$ iff $\forall r, s \in R$: $r.\alpha = s.\alpha \Rightarrow r.\beta = s.\beta$
- (There is a function f: X $D_\alpha \rightarrow$ X D_β)

R			
A	*B*	*C*	*D*
a4	b2	c4	d3
a1	b1	c1	d1
a1	b1	c1	d2
a2	b2	c3	d2
a3	b2	c4	d3

$\{A\} \rightarrow \{B\}$

$\{C, D\} \rightarrow \{B\}$

Not: $\{B\} \rightarrow \{C\}$

Convention:

$CD \rightarrow B$

Example

Family Tree				
Child	Father	Mother	Grandma	Grandpa
Sofie	Alfons	Sabine	Lothar	Linde
Sofie	Alfons	Sabine	Hubert	Lisa
Niklas	Alfons	Sabine	Lothar	Linde
Niklas	Alfons	Sabine	Hubert	Lisa
…	…	…	Lothar	Martha
...	...	...	...	...

Example

Family Tree				
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Sofie	Alfons	Sabine	Lothar	Linde
Sofie	Alfons	Sabine	Hubert	Lisa
Niklas	Alfons	Sabine	Lothar	Linde
Niklas	Alfons	Sabine	Hubert	Lisa
...	...	...	Lothar	Martha
...	...	...	...	...

- Child→ Father,Mother
- Child, Grandpa → Grandma
- Child, Grandma → Grandpa

Analogy to functions

- f1 : Child → Father
  - E.g., f1(Niklas) = Alfons
- f2: Child → Mother
  - E.g., f2(Niklas) = Sabine
- f3: Child x Grandpa → Grandma
- FD: Child → Father, Mother
  - represents two functions (f1, f2)
  - Komma on right side indicates multiple functions
- FD: Child, Grandpa → Grandma
  - Komma on the left side indicates Cartesian product

Keys

- $\alpha \subseteq \mathcal{R}$ is a superkey iff
  - $\alpha \rightarrow \mathcal{R}$
- $\alpha \rightarrow \beta$ is minimal iff
  - $\forall A \in \alpha: \neg((\alpha - \{A\}) \rightarrow \beta)$
- Notation for minimal functional dependencies: $\alpha \rightarrow^{\cdot} \beta$
- $\alpha \subseteq \mathcal{R}$ is a key (or candidate key) iff
  - $\alpha \rightarrow^{\cdot} \mathcal{R}$

Determining Keys

Town			
Name	Canton	AreaCode	Population
Buchs	AG	081	6500
Buchs	SG	071	8000
Zurich	ZH	044	300000
Lausanne	VD	021	60000
...	...	...	...

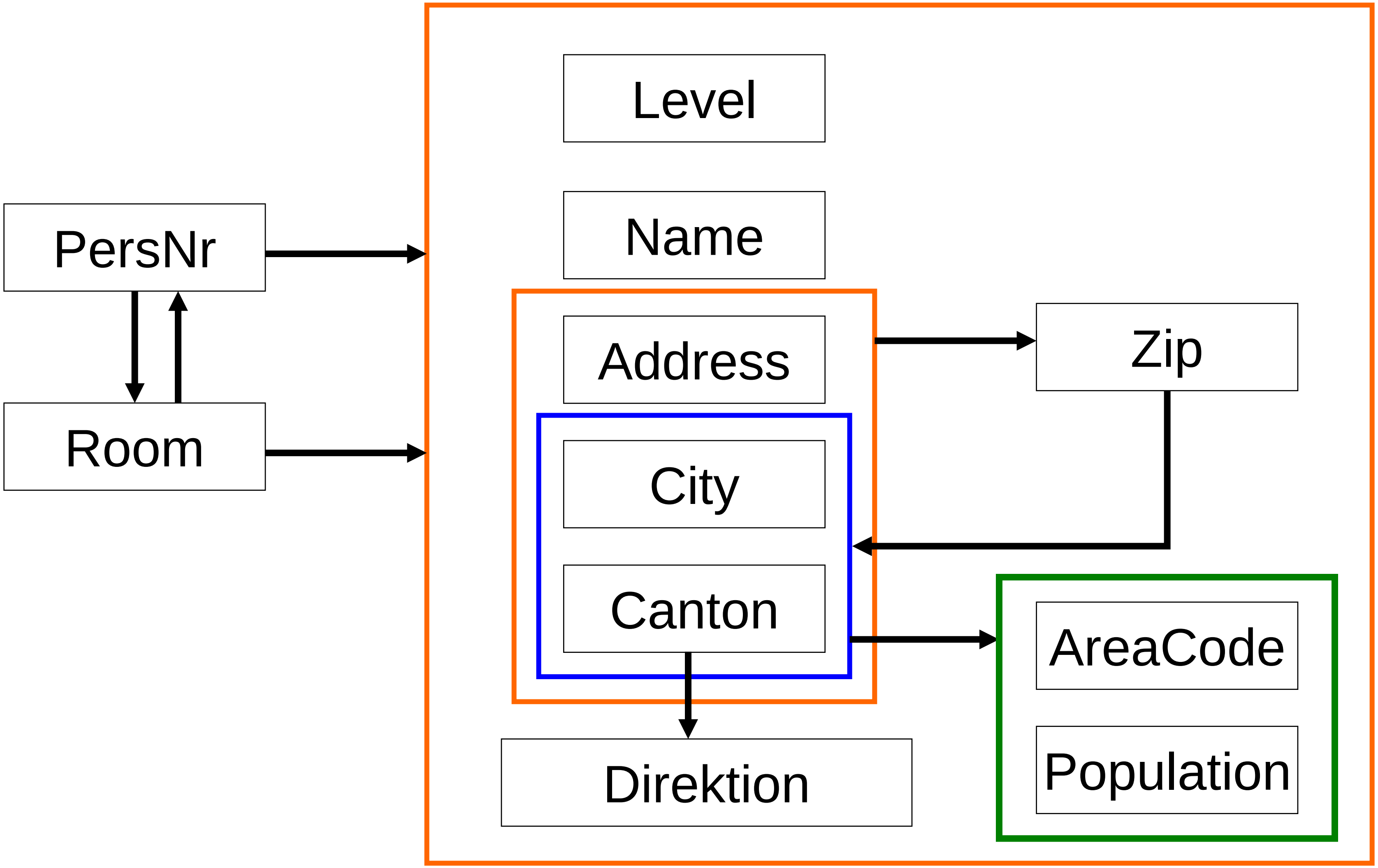
- Keys of Town
  - {Name, Canton}
  - {Name, AreaCode}

- N.B. Two small towns may have the same area code.

Determining Functional Dependencies

- Professor: {[PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion]}
  - {PersNr} → {PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion}
  - {City, Canton} → {Population, AreaCode}
  - {Zip} → {Canton, City, Population}
  - {Canton, City, Address} → {Zip}
  - {Canton} → {Direktion}
  - {Room} → {PersNr}
- Additional functional dependencies (inferred):
  - {Room} → {PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion}
  - {Zip} → {Direktion}

Visualization of Funct. Dependencies



Armstrong Axioms: Inference of FDs

- Reflexivity
  - $(\beta \subseteq \alpha) \Rightarrow \alpha \rightarrow \beta$
  - Special case: $\alpha \rightarrow \alpha$

- Augmentation
  - $\alpha \rightarrow \beta \Rightarrow \alpha\gamma \rightarrow \beta\gamma$.
  - (Notation $\alpha\gamma := \alpha \cup \gamma$)

- Transitivity
  - $\alpha \rightarrow \beta \wedge \beta \rightarrow \gamma \Rightarrow \alpha \rightarrow \gamma$.

- These three axioms are complete. All possible other rules can be implied from these axioms.

Other rules

- Union of FDs:
  - $\alpha \to \beta \wedge \alpha \to \gamma \Rightarrow \alpha \to \beta\gamma$
- Decomposition:
  - $\alpha \to \beta\gamma \Rightarrow \alpha \to \beta \wedge \alpha \to \gamma$
- Pseudo transitivity:
  - $\alpha \to \beta \wedge \gamma\beta \to \delta \Rightarrow \alpha\gamma \to \delta$

Correctness of Union rule

- Premise: $\alpha \to \beta$ and $\alpha \to \gamma$
- Claim: $\alpha \to \beta\gamma$
- Proof:
  1. $\alpha \to \beta$ (Premise)
  2. $\alpha\gamma \to \beta\gamma$ (Augmentation)
  3. $\alpha \to \gamma$ (Premise)
  4. $\alpha \to \alpha\gamma$ (Augmentation)
  5. $\alpha \to \beta\gamma$ (Transitivity of (4) and (2)) qed

Closure of Attributes

- Input:
  - F: a set of FDs
  - α: a set of attributes
- Output: $\alpha+$ such that $\alpha \rightarrow \alpha+$

```
Closure(F, α)
   result := α                                          // Reflexivity
   while   (result has changed) do
        foreach FD: β → γ in F do                       // Transitivity
                if  β ⊆ result then result := result ∪ γ
   output(result)
```

- Exercise: Proof that Closure is deterministic and terminates.

Example: Closure of ZIP (Slide 8)

Minimal Basis

Fc is a minimal basis of F iff:

1. $Fc \equiv F$
   - The closure of all attribute set is the same in Fc and F

2. All functional dependencies in Fc are minimal:
   - $\forall A \in \alpha: (Fc - (\alpha \to \beta) \cup ((\alpha - \{A\}) \to \beta)) \not\equiv Fc$
   - $\forall B \in \beta: (Fc - (\alpha \to \beta) \cup (\alpha \to (\beta - \{B\}))) \not\equiv Fc$

3. In Fc, there are no two functional dependencies with the same left side.
   - Can be achieved by applying the Union rule.

Computing the Minimum Basis

1. Reduction of left sides of FDs. Let $\alpha \rightarrow \beta \in F$, $A \in \alpha$:

   if $\beta \subseteq$ Closure(F, α – A)

   then replace $\alpha \rightarrow \beta$ **with** (α - A) $\rightarrow \beta$ in F

2. Reduction of right sides of FDs. Let $\alpha \rightarrow \beta \in F$, $B \in \beta$:

   if B $\in$ Closure(F – ($\alpha \rightarrow \beta$) $\cup$ ($\alpha \rightarrow (\beta - B)$), α)

   then replace $\alpha \rightarrow \beta$ **with** $\alpha \rightarrow (\beta – B)$ in F

1. Remove FDs: $\alpha \rightarrow \varnothing$ (clean-up of Step 2)

2. Apply Union rule to FDs with the same left side.

Determining Functional Dependencies

- Professor: {[PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion]}
  - {PersNr} → {PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion}
  - {City, Canton} → {Population, AreaCode}
  - {Zip} → {Canton, City, Population}
  - {Canton, City, Address} → {Zip}
  - {Canton} → {Direktion}
  - {Room} → {PersNr}
- Additional functional dependencies (inferred):
  - {Room} → {PersNr, Name, Level, Room, City, Address, Zip, AreaCode, Canton, Population, Direktion}
  - {Zip} → {Direktion}

Correctness of the Algorithm (Left Reduction)

- Premise: $\beta \subseteq$ Closure(F, α - A)
- Claim: Closure(F-$\{\alpha \rightarrow \beta\} \cup \{ (\alpha - A) \rightarrow \beta \}$, α - A) $\subseteq$ Closure(F, α - A)
- Proof:

let $\gamma \in$ Closure(F-$\{\alpha \rightarrow \beta\} \cup \{ (\alpha - A) \rightarrow \beta \}$, α - A)
$\gamma \in$ Closure(F, α - A $\cup$ β) (Apply FD $(\alpha - A) \rightarrow \beta$)
$\gamma \in$ Closure(F, α - A) (Premise) qed

Bad Schemas

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- Update-Anomaly
  - What happens when Sokrates moves to a different room?
- Insert-Anomaly
  - What happens if Roscoe is elected as a new professor?
- Delete-Anomaly
  - What happens if Popper does not teach this semester?

Decomposition of Relations

- Bad relations combine several concepts
  - decompose them so that each concept in one relation
  - $\mathcal{R} \rightarrow \mathcal{R}_1, \ldots, \mathcal{R}_n$

1. Lossless Decomposition

$$\mathcal{R} = \mathcal{R}_1 \text{ A } \mathcal{R}_2 \text{ A } \ldots \text{ A } \mathcal{R}_n$$

2. Preservation of Dependencies

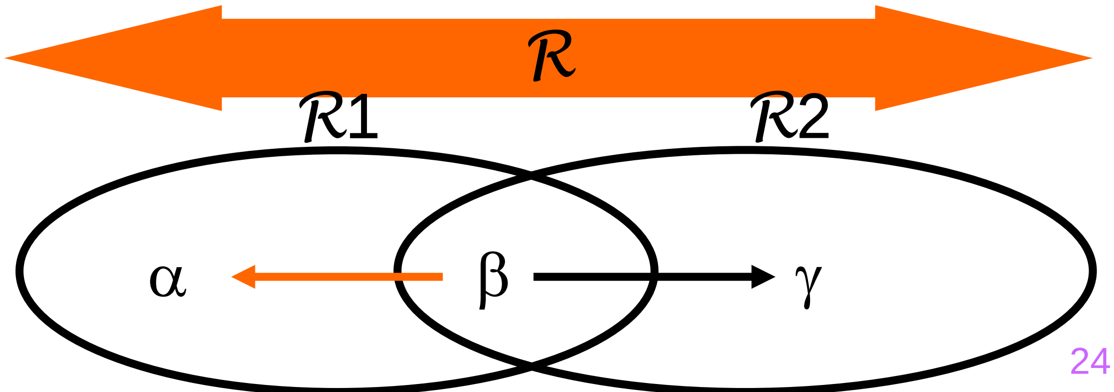
   - $FD(\mathcal{R})+ = (FD(\mathcal{R}_1) \cup \ldots \cup FD(\mathcal{R}_n))+$

When is a decomposition lossless?

- Let $\mathcal{R} = \mathcal{R}1 \cup \mathcal{R}2$
  - R1 := $\Pi_{\mathcal{R}1}$ (R)
  - R2 := $\Pi_{\mathcal{R}2}$ (R)

- Lemma: The decomposition is lossless if
  - $(\mathcal{R}1 \cap \mathcal{R}2) \rightarrow \mathcal{R}1$ or
  - $(\mathcal{R}1 \cap \mathcal{R}2) \rightarrow \mathcal{R}2$

- Exercise: Proof of this Lemma.



Example

Drinker		
Pub	*Guest*	*Beer*
Kowalski	Kemper	Pils
Kowalski	Eickler	Hefeweizen
Innsteg	Kemper	Hefeweizen

Lossy Decomposition

Drinker		
Pub	*Guest*	*Beer*
Kowalski	Kemper	Pils
Kowalski	Eickler	Hefeweizen
Innsteg	Kemper	Hefeweizen

$\Pi_{\text{Pub, Guest}}$

$\Pi_{\text{Guest, Beer}}$

Visitor	
Pub	*Guest*
Kowalski	Kemper
Kowalski	Eickler
Innsteg	Kemper

Drinks	
Guest	*Beer*
Kemper	Pils
Eickler	Hefeweizen
Kemper	Hefeweizen

Drinker		
Kneipe	*Gast*	*Bier*
Kowalski	Kemper	Pils
Kowalski	Eickler	Hefeweizen
Innsteg	Kemper	Hefeweizen

$\Pi_{....}$

Visitor	
Pub	*Guest*
Kowalski	Kemper
Kowalski	Eickler
Innsteg	Kemper

Drinks	
Guest	*Beer*
Kemper	Pils
Eickler	Hefeweizen
Kemper	Hefeweizen

A

$\neq$

Visitor A Drinks		
Pub	*Guest*	*Beer*
Kowalski	Kemper	Pils
Kowalski	Kemper	Hefeweizen
Kowalski	Eickler	Hefeweizen
Innsteg	Kemper	Pils
Innsteg	Kemper	Hefeweizen

Comments on the Example

- Drinker has one (non-trivial) functional dependency
  - $\{Pub, Guest\} \rightarrow \{Beer\}$
- But none of the criteria of the Lemma hold
  - $\{Guest\} \nrightarrow \{Beer\}$
  - $\{Guest\} \nrightarrow \{Pub\}$
- The problem is that Kemper likes different beer in different pubs.

Lossless Decomposition

Parents		
Father	*Mother*	*Child*
Johann	Martha	Else
Johann	Maria	Theo
Heinz	Martha	Cleo

$\Pi_{\text{Father, Child}}$

$\Pi_{\text{Mother, Child}}$

Father	
Father	*Child*
Johann	Else
Johann	Theo
Heinz	Cleo

Mother	
Mother	*Child*
Martha	Else
Maria	Theo
Martha	Cleo

Comments on Example

- Parents: {[Father, Mother, Child]}
- Father: {[Father, Child]}
- Mother: {[Mother, Child]}

- Actually, both criteria of the lemma are met:
  - {Child}$\rightarrow${Mother}
  - {Child}$\rightarrow${Father}

- {Child} is a key of all three relations
  - Wrt loss of info, it never hurts to decompose with a key
  - However, it is never beneficial either. Why?

Preservation of Dependencies

- Let $\mathcal{R}$ be decomposed into $\mathcal{R}1, \ldots, \mathcal{R}n$
- $F_{\mathcal{R}} = (F_{\mathcal{R}1} \cup \ldots \cup F_{\mathcal{R}n})$

- ZipCodes: {[Street, City, Canton, Zip]}

- Functional dependencies in ZipCodes
  - {Zip} → {City, Canton}
  - {Street, City, Canton} → {Zip}

- What about this decomposition?
  - Streets: {[Zip, Street]}
  - Cities: {[Zip, City, Canton]}

- Is it lossless? Does it preserve functional depend.?

Decomposition of ZipCodes

ZipCodes			
City	***Canton***	***Street***	*Zip*
Buchs	AG	Goethestr.	5033
Buchs	AG	Schillerstr.	5034
Buchs	SG	Goethestr.	8107

$\Pi_{\text{Zip,Street}}$

$\Pi_{\text{City,Canton,Zip}}$

Streets	
Zip	***Street***
8107	Goethestr.
5033	Goethestr.
5034	Schillerstr.

Cities		
City	*Canton*	***Zip***
Buchs	AG	5033
Buchs	AG	5034
Buchs	SG	8107

{Street, City, Canton} → {Zip} not checkable in decomp. schema

It is possible to insert inconsistent tuples

Violation of City,Canton,Street→Zip

ZipCodes			
City	***Canton***	***Street***	*Zip*
Buchs	AG	Goethestr.	5033
Buchs	AG	Schillerstr.	5034
Buchs	SG	Goethestr.	8107

$\Pi_{\text{Zip,Street}}$

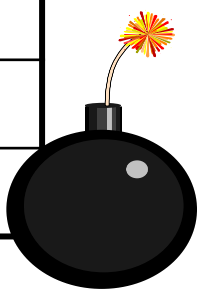
$\Pi_{\text{City,Canton,Zip}}$

Streets	
Zip	***Street***
8107	Goethestr.
5033	Goethestr.
5034	Schillerstr.
8108	Goethestr.

Cities		
City	*Canton*	***Zip***
Buchs	AG	5033
Buchs	AG	5034
Buchs	SG	8107
Buchs	SG	8108

Violation of City,Canton,Street→Zip

ZipCodes			
City	***Canton***	***Street***	*Zip*
Buchs	AG	Goethestr.	5033
Buchs	AG	Schillerstr.	5034
Buchs	SG	Goethestr.	8107
Buchs	SG	Goethestr.	8108



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Streets	
Zip	***Street***
8107	Goethestr.
5033	Goethestr.
5034	Schillerstr.
8108	Goethestr.

Cities		
City	*Cantion*	***Zip***
Buchs	AG	5033
Buchs	AG	5034
Buchs	SG	8107
Buchs	SG	8108

First Normal Form

- Only atomic domains (as in SQL 92)

Parents		
Father	*Mother*	*Children*
Johann	Martha	{Else, Lucie}
Johann	Maria	{Theo, Josef}
Heinz	Martha	{Cleo}

vs.

Parents		
Father	*Mother*	*Child*
Johann	Martha	Else
Johann	Martha	Lucie
Johann	Maria	Theo
Johann	Maria	Josef
Heinz	Martha	Cleo

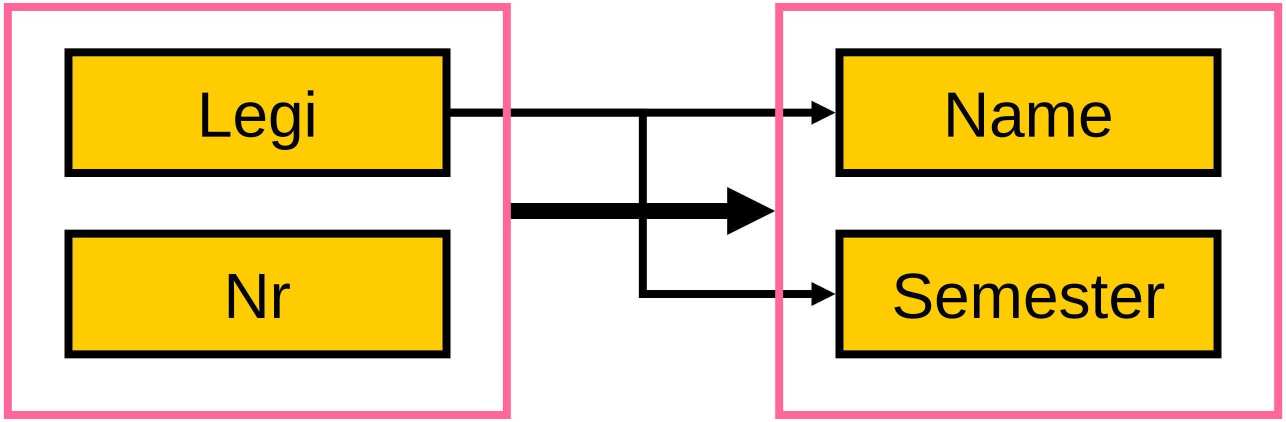
Second Normal Form

- $\mathcal{R}$ is in 2NF iff every non-key attribute is minimally dependent on every key.

StundentAttends			
Legi	**Nr**	Name	Semester
26120	5001	Fichte	10
27550	5001	Schopenhauer	6
27550	4052	Schopenhauer	6
28106	5041	Carnap	3
28106	5052	Carnap	3
28106	5216	Carnap	3
28106	5259	Carnap	3
...	...	...	...

- StudentAttends is not in 2NF!!!
  - {Legi} → {Name, Semester}

Second Normal Form



- Insert Anomaly: What about students who attend no lecture?
- Update Anomaly: Promotion of Carnab to the 4th semester.
- Delete Anomaly: Fichte drops his last course?

- Solution: Decompose into two relations
  - attends: {[Legi, Nr]}
  - Student: {[Legi, Name, Semester]}

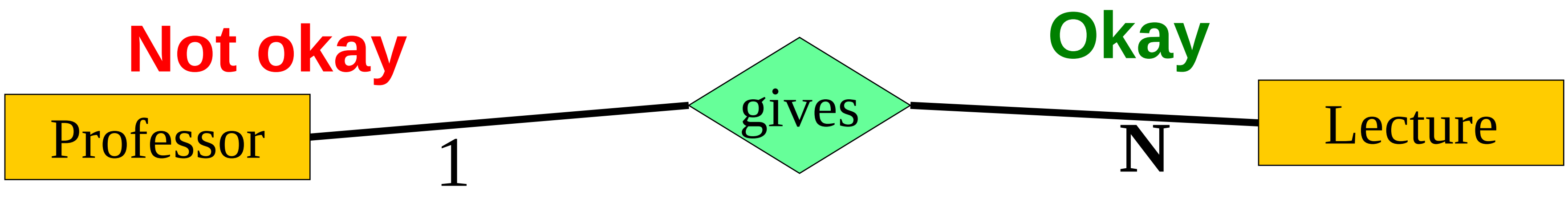
- Student, attends are in 2NF. The decompostion is lossless and preserves dependencies.

2NF and ER Modelling

- Violation of 2NF
  - mixing an entity with an N:M (or 1:N) relationship
  - E.g., mixing Student (entity) with attends (N:M)

- Solution
  - Separate: entity and relationship
  - i.e., implement entity and relationship in separate relations

- However, okay to mix entity and 1:1/N:1 relationship

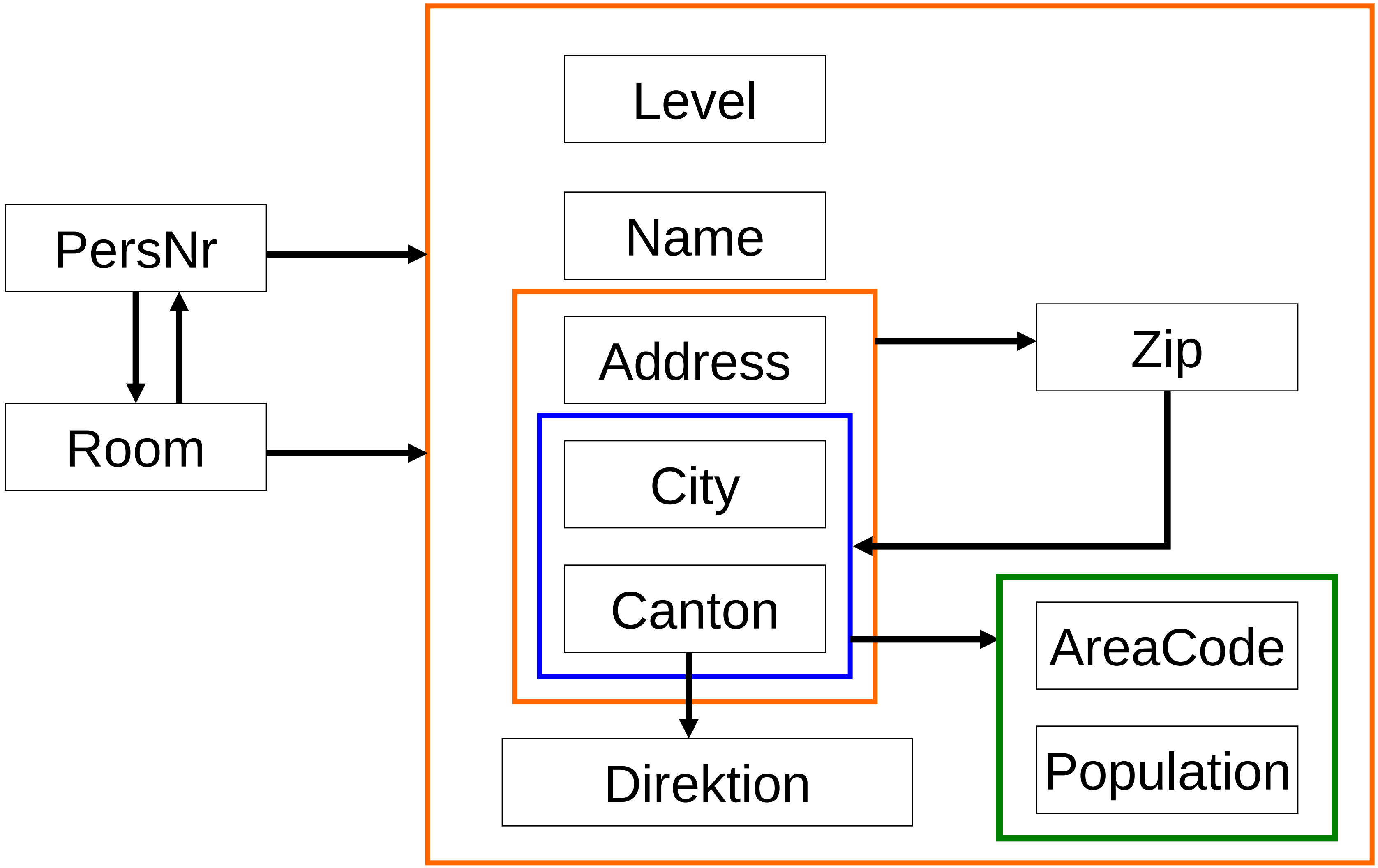


Third Normal Form

- $\mathcal{R}$ is in 3NF iff for all $\alpha \rightarrow B$ in $\mathcal{R}$ at least one condition holds:
  - B $\in \alpha$ (i.e., $\alpha \rightarrow B$ is trivial)
  - B is an attribute of at least one key
  - α is a superkey of $\mathcal{R}$

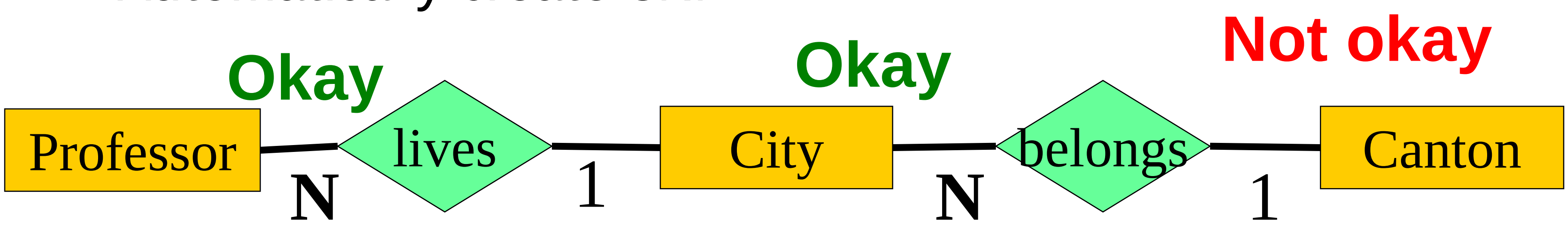
- If $\alpha \rightarrow B$ does not fulfill any of these conditions
  - α is a concept in its own right.

Example: 2NF but not 3NF



3NF and ER Modelling

- Violation of 3NF
  - mixing several entities (maybe connected by relationships)
  - e.g., Professor, City, Canton
- Solution
  - implement each entity in a separate relation
  - (implement N:M relationships in separate relation)
- ER Modelling and Rules of ER -> relational
  - Automatically create 3NF



3NF implies 2NF

- Premise: $\mathcal{R}$ is in 3NF
- Claim: $\mathcal{R}$ is in 2NF
- Proof:
  - assume $\mathcal{R}$ is not in 2NF
  - By definition of 2NF: exists $\alpha \to \mathrm{B}$ such that
    - (1) B is not part of any key
    - (2) $\alpha \subseteq \kappa$, κ is a key
  - $\alpha \to \mathrm{B}$ is evil
    - it is not trivial (otherwise B would be part of a key)
    - B is not part of any key (1)
    - α is not a superkey (2)
  - $\mathcal{R}$ is not in 3NF. qed

Synthesis Algorithm

- Input: Relation $\mathcal{R}$, FDs F
- Output: $\mathcal{R}$1, …, $\mathcal{R}$n such that
  - $\mathcal{R}$1, …, $\mathcal{R}$n is a lossless decomposition of $\mathcal{R}$.
  - $\mathcal{R}$1, …, $\mathcal{R}$n preserves dependencies.
  - All $\mathcal{R}$1, …, $\mathcal{R}$n are in 3NF.

Synthesis Algorithm

1. Compute the minimal basis Fc of F.

2. For all $\alpha \to \beta \in$ Fc create:
   - $\mathcal{R}\alpha := \alpha \cup \beta$

3. If exists $\kappa \subseteq \mathcal{R}$ such that κ is a key of $\mathcal{R}$ create:
   - $\mathcal{R}\kappa := \kappa$
   - (N.B.: $\mathcal{R}\kappa$ has no non-trivial functional dependencies.)

4. Eliminate $\mathcal{R}\alpha$ if exists $\mathcal{R}\alpha$` such that:
   - $\mathcal{R}\alpha \subseteq \mathcal{R}\alpha$`

Example: Synthesis Algorithm

- Professor: {[PersNr, Name, Level, Room, City, Street, Zip, AreaCode, Canton, Population, Direktion]}
  1. {PersNr} → {Name, Level, Room, Canton, Street, Canton}
  2. {Room} → {PersNr}
  3. {Street, Canton, City} → {Zip}
  4. {City, Canton} → {Population, AreaCode}
  5. {Canton} → {Direktion}
  6. {Zip} → {Canton, City}

- Professor: {[PersNr, Name, Level, Room, City, Street, Canton]}
- ZipCodes: {[Street, Canton, City, Zip]}
- Cities: {[City, Canton, Population, AreaCode]}
- Administration: {[Canton, Direktion]}

Example why Step 3 is needed

- StudentAttends(**Legi, Nr,** Name, Semester)

- Minimum Basis (Step 1)
  - {Legi} $\rightarrow$ {Name, Semester}

- Relation generated from minimum basis (Step 2)
  - Student(**Legi**, Name, Semester)

- Relation generated from Step 3
  - attends(**Legi, Nr**)

- The attends relation is needed!

Corner Case: Step 3

- R(A, B, C, D)
  - B -> C, D
  - D -> B
- Keys of R
  - A, B
  - A, D
- Decomposition into 3NF (Synthesis Algorithm)
  - R1(B, C, D)
  - R2(A, B)
- N.B. R3(A,D) is not needed!!!
  - Needs to be cleaned up in Step 4!

ZipCodes(Street, Canton, City, Zip)

- Is ZipCodes in 3NF?
  - Keys: {Street,Canton,City}, {Zip,Street}
  - All attributes are part of keys. There are no evil FDs!

- Does the decomposition preserve dependencies?
  - Yes!

- Is the decomposition lossless?
  - Professor $\cap$ ZipCodes = {Street,Canton,City}
  - {Street,Canton,City} $\rightarrow$ ZipCodes
  - Criterion of Lemma is fullfilled!

- Is ZipCode free of redundancy?

Exercises

- Proof for the following lemmas:
  - The synthesis algorithm preserves dependencies.
  - The synthesis algorithm creates lossless decompositions.
  - The synthesis algorithm creates relations in 3NF only.
  - The synthesis algorithm creates relations in 2NF only.

Synthesis Algo produces 3NF only

- Let $\mathcal{R}_i$ be a relation created by the Synthesis Algo
- Case 1: $\mathcal{R}_i$ was created in Step 3 of the algo
  - $\mathcal{R}_i$ contains a key of $\mathcal{R}$
  - there are no non-trivial FDs in $\mathcal{R}_i$
  - $\mathcal{R}_i$ is in 3NF
- Case 2: $\mathcal{R}_i$ was created in Step 2 by an FD: $\alpha \rightarrow \beta$
  - (1) $\mathcal{R}_i := \alpha \cup \beta$
  - (2) α is a key of $\mathcal{R}_i$
    - α is minimal because of left reduction of minimal basis
    - $\alpha \rightarrow \mathcal{R}_i$ by construction of $\mathcal{R}_i$
  - (3) $\alpha \rightarrow \beta$ is not evil because α is a superkey of $\mathcal{R}_i$
  - (4) Let $\gamma \rightarrow \delta$ be any <u>other</u> non-trivial FD ($\gamma \rightarrow \delta \neq \alpha \rightarrow \beta$)
    - $\delta \subseteq \alpha$ because of right reduction in minimal basis and because $\alpha \rightarrow \gamma$
    - δ contains only attributes of a key; $\gamma \rightarrow \delta$ is not evil qed

Boyce-Codd-Normal Form (BCNF)

- $\mathcal{R}$ is in BCNF iff for all $\alpha \rightarrow \mathrm{B}$ in $\mathcal{R}$ at least one condition holds:
  - B $\in \alpha$ (i.e., $\alpha \rightarrow \mathrm{B}$ is trivial)
  - α is a superkey of $\mathcal{R}$

- $\mathcal{R}$ in BCNF implies $\mathcal{R}$ in 3NF
  - Proof trivial from definition

- Result
  - any schema can be decomposed losslessly into BCNF
  - but, preservation of dependencies cannot be guaranteed
  - need to trade „correctness“ for „efficiency“
  - that is why 3NF is so important in practice

ZipCodes(Street, Canton, City, Zip)

- ZipCodes is not in BCNF
  - {Zip} → {Canton, City} // evil
  - {Street, Canton, City} → {Zip} // okay
- Redundancy in ZipCodes
  - (Rämistr., Zürich, Zürich, 8006)
  - (Universitätsstr., Zürich, Zürich, 8006)
  - (Schmid-Str., Zürich, Zürich, 8006)
  - stores several times that 8006 belongs to Zürich
- Exercise: How would you model ZipCodes in ER?
  - What would the relational schema look like?

Decomposition Algorithm (BCNF)

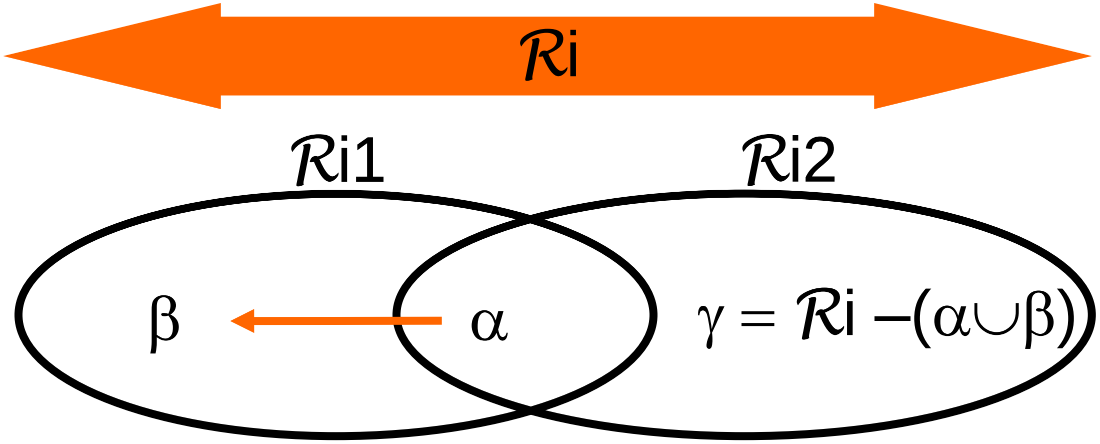
- Input: $\mathcal{R}$
- Output: $\mathcal{R}1, \ldots, \mathcal{R}n$ such that
  - $\mathcal{R}1, \ldots, \mathcal{R}n$ is a lossless decomposition of $\mathcal{R}$.
  - $\mathcal{R}1, \ldots, \mathcal{R}n$ are in BCNF.
  - (Preservation of dependencies is not guaranteed.)

Decomposition Algorithm

- Input: $\mathcal{R}$
- Output: $\mathcal{R}1, \ldots, \mathcal{R}n$

result = $\{\mathcal{R}\}$
while ($\exists$ $\mathcal{R}$i $\in$ Z: $\mathcal{R}$i is not in BCNF))
    let $\alpha \rightarrow \beta$ be evil in $\mathcal{R}$i
    $\mathcal{R}$i1 = $\alpha \cup \beta$
    $\mathcal{R}$i2 = $\mathcal{R}$i - β
    result = (result – $\{\mathcal{R}$i$\}$) $\cup$ $\{\mathcal{R}$i1$\}$ $\cup$ $\{\mathcal{R}$i2$\}$
output(result)

Visualization of Decomposition Algo



Decomposition of ZipCodes

- ZipCodes: {[Street, City, Canton, Zip]}
  - {Zip} → {City, Canton}   // evil
  - {Street, City, Canton} → {Zip}   // okay

- Applying the decomposition algorithm…
  - Street: {[Zip, Street]}
  - Cities: {[Zip, City, Canton]}

- Assessment
  - decomposition is lossless
  - decomposition does not preserve dependencies

Cities is not in BCNF

- Cities: {[City, Canton, Direktion, Population]}
- FDs of Cities:
  - {City, Canton} → {Population}
  - {Canton} → {Direktion}
  - {Direktion} → {Canton}
- Keys:
  - {City, Canton}

Köln	NRW	Rütgers	1mio
Bonn	NRW	Rütgers	200K
Aachen	NRW	Rütgers	200K

- In which highest NF is Cities?
  - N.B. decomposition algo can also be applied to non 3NF!

Decomposition of Cities

- Cities: {[City, Canton, Direktion, Population]}
  - {Canton} → {Direktion} // evil
  - {Direktion} → {Canton} // evil
  - {City, Canton} → {Population} // okay

- $\mathcal{R}$i1:
  - Administration: {[Canton, Direktion]}

- $\mathcal{R}$i2:
  - Cities: {[City, Canton, Population]}

- Is this decomposition lossless? Preserves depend.?

Fourth Normal Form

Skills		
PersNr	Language	Programming
3002	Greek	C
3002	Latin	Pascal
3002	Greek	Pascal
3002	Latin	C
3005	German	Ada

- Give FDs, keys. Which highest normal form?
- Does „Skills“ have redundancy?
  - What happens if 3002 learns a new language?
- How would you model Skills in ER? How translated?

NFNF

Skills		
PersNr	Language	Programming
3002	{Greek, Latin}	{C, Pascal}
3005	{German}	{Ada}

- This model would work: no redundancy, no anomolies
- But, unfortunately, not implementable in SQL 2
- It is implementable in SQL 3 and XML
  - That is why design theory needs to be adapted to XML

What is wrong with this?

Skills		
PersNr	Language	Programming
3002	Greek	C
3002	Latin	Pascal
3005	German	Ada

- Who knows Greek and Pascal?
- Anomolies?
  - What happens if 3002 learns a new language?

Multi-value Dependencies

R		
A	B	C
a	b	c
a	bb	cc
a	bb	c
a	b	cc

- A →→ B
- A →→ C

Multi-value Dependencies (MVD)

R		
$\overbrace{A1 \ldots Ai}^{\alpha}$	$\overbrace{Ai{+}1 \ldots Aj}^{\beta}$	$\overbrace{Aj{+}1 \ldots An}^{\gamma}$
a1 … ai	ai+1 … aj	aj+1 … an
a1 … ai	bi+1 … bj	bj+1 … bn
a1 … ai	bi+1 … bj	aj+1 … an
a1 … ai	ai+1 … aj	bj+1 … bn

- $\alpha \rightarrow\rightarrow \beta$ iff
  - $\forall$ t1, t2 $\in$ R: t1.α = t2.α $\Rightarrow$ $\exists$ t3, t4 $\in$ R:
    - t3.α = t4.α = t1.α = t2.α
    - t3.β = t1.β, t4.β = t2.β
    - t3.γ = t2.γ , t4.γ = t1.γ

MVD: Example

Skills		
PersNr	**Language**	**Programming**
3002	Greek	C
3002	Latin	Pascal
3002	Greek	Pascal
3002	Latin	C
3005	German	Ada

- MVDs of Skills
  - {PersNr}→→{Language}
  - {PersNr}→→{Programming}
  - {Language} {PersNr, Programming} (???)
- MVDs can result in anomalies and redundancy

Are MVDs symmetric?

- NO!
  - NOT {Language}$\rightarrow\rightarrow${PersNr}

Skills		
PersNr	Language	Programming
3002	Greek	C
3002	Latin	Pascal
3002	Greek	Pascal
3002	Latin	C
3005	German	Ada
3007	Greek	XQuery

- Exercise: Find examples for symmetric MVDs!

MVD: Example

Skills		
PersNr	Language	Programming
3002	Greek	C
3002	Latin	Pascal
3002	Greek	Pascal
3002	Latin	C
3005	German	Ada

$\Pi_{\text{PersNr, Language}}$

$\Pi_{\text{PersNr, Programming}}$

Language	
PersNr	Language
3002	Greek
3002	Latin
3005	German

Programming	
PersNr	Programming
3002	C
3002	Pascal
3005	Ada

MVD: Example

Skills		
PersNr	Language	Programming
3002	Greek	C
3002	Latin	Pascal
3002	Greek	Pascal
3002	Latin	C
3005	German	Ada

Join

Language	
PersNr	Language
3002	Greek
3002	Latin
3005	German

Programming	
PersNr	Programming
3002	C
3002	Pascal
3005	Ada

Example: Drinkers

- drinkers: {[name, addr, phones, beersLiked]}
  - some people have several phones
  - some people like several kind of beers

- FDs and MVDs
  - name $\rightarrow$ addr
  - name $\rightarrow\rightarrow$ phones
  - name $\rightarrow\rightarrow$ beersLiked

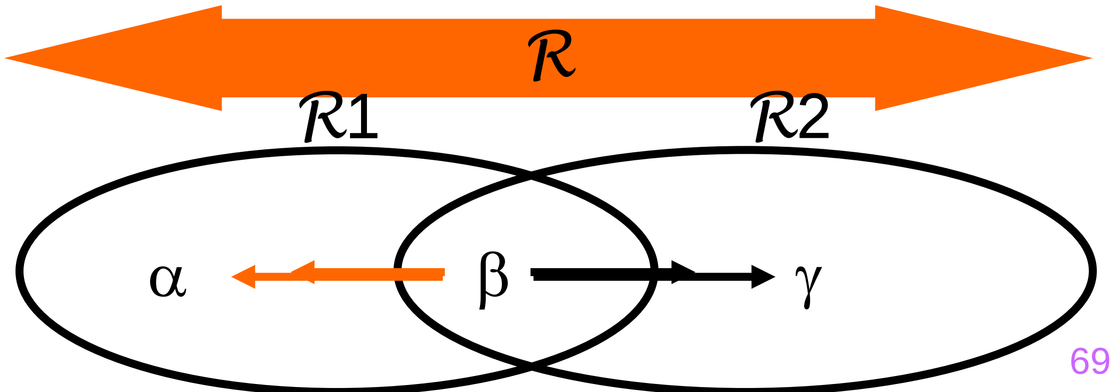
- Again, MVDs indicate redundancy
  - phones and beersLiked are independent concepts
  - (Would work if you could store them as sets: NFNF.)

Lossless Decompositions with MVDs

- Let $\mathcal{R} = \mathcal{R}1 \cup \mathcal{R}2$
  - R1 := $\Pi_{\mathcal{R}1}$ (R)
  - R2 := $\Pi_{\mathcal{R}2}$ (R)

- Lemma: The decomposition is lossless **iff**
  - ($\mathcal{R}1 \cap \mathcal{R}2$) $\rightarrow\rightarrow$ $\mathcal{R}1$ or
  - ($\mathcal{R}1 \cap \mathcal{R}2$) $\rightarrow\rightarrow$ $\mathcal{R}2$

- Exercise: Proof of this Lemma.
  Which direction is easier?



Laws of MVDs

- Trivial MVDs: $\alpha \twoheadrightarrow \mathcal{R}$
  - Check criterion of MVDs ($\beta = \mathcal{R}$, $\gamma = \emptyset$)
    - $\forall$ t1, t2 $\in$ R: t1.α = t2.$\alpha \Rightarrow \exists$ t3, t4 $\in$ R:
      - t3.α = t4.α = t1.α = t2.α
      - t3.β = t1.β, t4.β = t2.β
      - t3.γ = t2.γ , t4.γ = t1.γ
  - **let** t1, t2 $\in$ R: t1.α = t2.α
  - **set** t3=t1, t4=t2
    - t3.α = t4.α = t1.α = t2.α (by def. of t3, t4)
    - t3.β = t1.β (by def. of t3)
    - t4.β = t2.β (by def. of t4)
    - t3.γ = t2.γ ($\gamma = \emptyset$)
    - t4.γ = t1.γ ($\gamma = \emptyset$) qed
- Adapt proof for: $\alpha \twoheadrightarrow \mathcal{R} - \alpha$

Laws of MVDs

- Promotion: $\alpha \rightarrow \beta \Rightarrow \alpha \rightarrow\rightarrow \beta$
  - **let** t1, t2 $\in$ R: t1.α = t2.α
    - (1) t1.β = t2.β ($\alpha \rightarrow \beta$)
  - **set** t3=t2, t4=t1
    - t3.α = t4.α = t1.α = t2.α (by def. of t3, t4)
    - t3.β = t1.β (1)
    - t4.β = t2.β (1)
    - t3.γ = t2.γ (by def. of t3)
    - t4.γ = t1.γ (by def. of t4) qed
- $\alpha \rightarrow\rightarrow \beta \Rightarrow \alpha \rightarrow \beta$ does not always hold!
  - e.g., PersNr $\rightarrow\rightarrow$ Language, but PersNr $\not\rightarrow$ Language

Laws of MVDs

- Reflexivity: $(\beta \subseteq \alpha) \Rightarrow \alpha \rightarrow \beta$
- Augmentation: $\alpha \rightarrow \beta \Rightarrow \alpha\gamma \rightarrow \beta\gamma$
- Transitivity: $\alpha \rightarrow \beta \wedge \beta \rightarrow \gamma \Rightarrow \alpha \rightarrow \gamma$
- Complement: $\alpha \rightarrow\rightarrow \beta \Rightarrow \alpha \rightarrow\rightarrow \mathcal{R} - \beta - \alpha$
- Multi-value Augmentation: $\alpha \rightarrow\rightarrow \beta \wedge (\delta \subseteq \gamma) \Rightarrow \alpha\gamma \rightarrow\rightarrow \beta\delta$
- Multi-value Transitivity: $\alpha \rightarrow\rightarrow \beta \wedge \beta \rightarrow\rightarrow \gamma \Rightarrow \alpha \rightarrow\rightarrow \gamma$
- Generalization (Promotion): $\alpha \rightarrow \beta \Rightarrow \alpha \rightarrow\rightarrow \beta$

Laws of MVDs (ctd.)

- Coalesce: $\alpha \rightarrow\rightarrow \beta \wedge (\gamma \subseteq \beta) \wedge (\delta \cap \beta = \emptyset) \wedge \delta \rightarrow \gamma \Rightarrow \alpha \rightarrow \gamma$
- Multi-value Union: $\alpha \rightarrow\rightarrow \beta \wedge \alpha \rightarrow\rightarrow \gamma \Rightarrow \alpha \rightarrow\rightarrow \beta\gamma$
- Intersection: $\alpha \rightarrow\rightarrow \beta \wedge \alpha \rightarrow\rightarrow \gamma \Rightarrow \alpha \rightarrow\rightarrow (\beta \cap \gamma)$
- Minus: $\alpha \rightarrow\rightarrow \beta \wedge \alpha \rightarrow\rightarrow \gamma \Rightarrow \alpha \rightarrow\rightarrow (\beta - \gamma) \wedge \alpha \rightarrow\rightarrow (\gamma - \beta)$
- Warning: **NOT** $(\alpha \rightarrow\rightarrow \beta\gamma \Rightarrow \alpha \rightarrow\rightarrow \beta \wedge \alpha \rightarrow\rightarrow \gamma)$
  - Not all rules of FDs apply to MVDs!
  - Exercise: find an example for which this law does not hold

WARNING

- **NOT** $(\alpha \to\to \beta\gamma \Rightarrow \alpha \to\to \beta \wedge \alpha \to\to \gamma)$
  - Not all rules of FDs apply to MVDs!
  - E.g., {Language} $\to\to$ {PersNr, Programming}
  - But, NOT {Language} $\to\to$ {PersNr}
  - And NOT {Language} $\to\to$ {Programming}

- Be careful with MVDs
  - Not a totally intuitive concept

Trivial MVDs

- $\alpha \to\to \beta$ is trivial iff
  - $\beta \subseteq \alpha$ or
  - $\beta = \mathcal{R} - \alpha$
- Proof: a trivial MVD holds for any relation.

Fourth Normal Form (4NF)

- $\mathcal{R}$ is in 4NF iff for all $\alpha \twoheadrightarrow \beta$ at least one condition holds:
  - $\alpha \twoheadrightarrow \beta$ is trivial
  - α is a superkey of $\mathcal{R}$

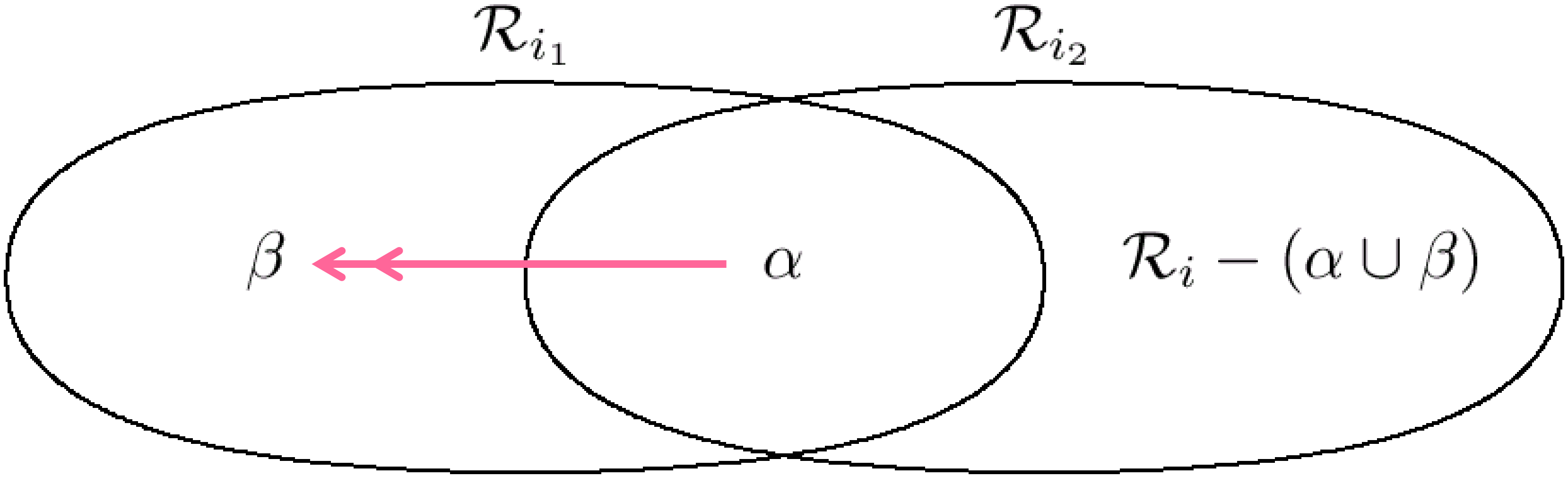
- $\mathcal{R}$ in 4NF implies $\mathcal{R}$ in BCNF
  - Proof is based on $\alpha \rightarrow \beta \Rightarrow \alpha \twoheadrightarrow \beta$.
  - (Can you prove that?)

Decomposition Algorithm for 4NF

- Input: $\mathcal{R}$
- Output: $\mathcal{R}1, \ldots, \mathcal{R}n$

result = {$\mathcal{R}$}
while ($\exists\ \mathcal{R}i \in Z$: $\mathcal{R}i$ is not in 4NF))
    let $\alpha \twoheadrightarrow \beta$ be evil in $\mathcal{R}i$
    $\mathcal{R}i1 = \alpha \cup \beta$
    $\mathcal{R}i2 = \mathcal{R}i - \beta$
    result = (result – {$\mathcal{R}i$}) $\cup$ {$\mathcal{R}i1$} $\cup$ {$\mathcal{R}i2$}
output(result)

Decomposition into 4 NF



Example

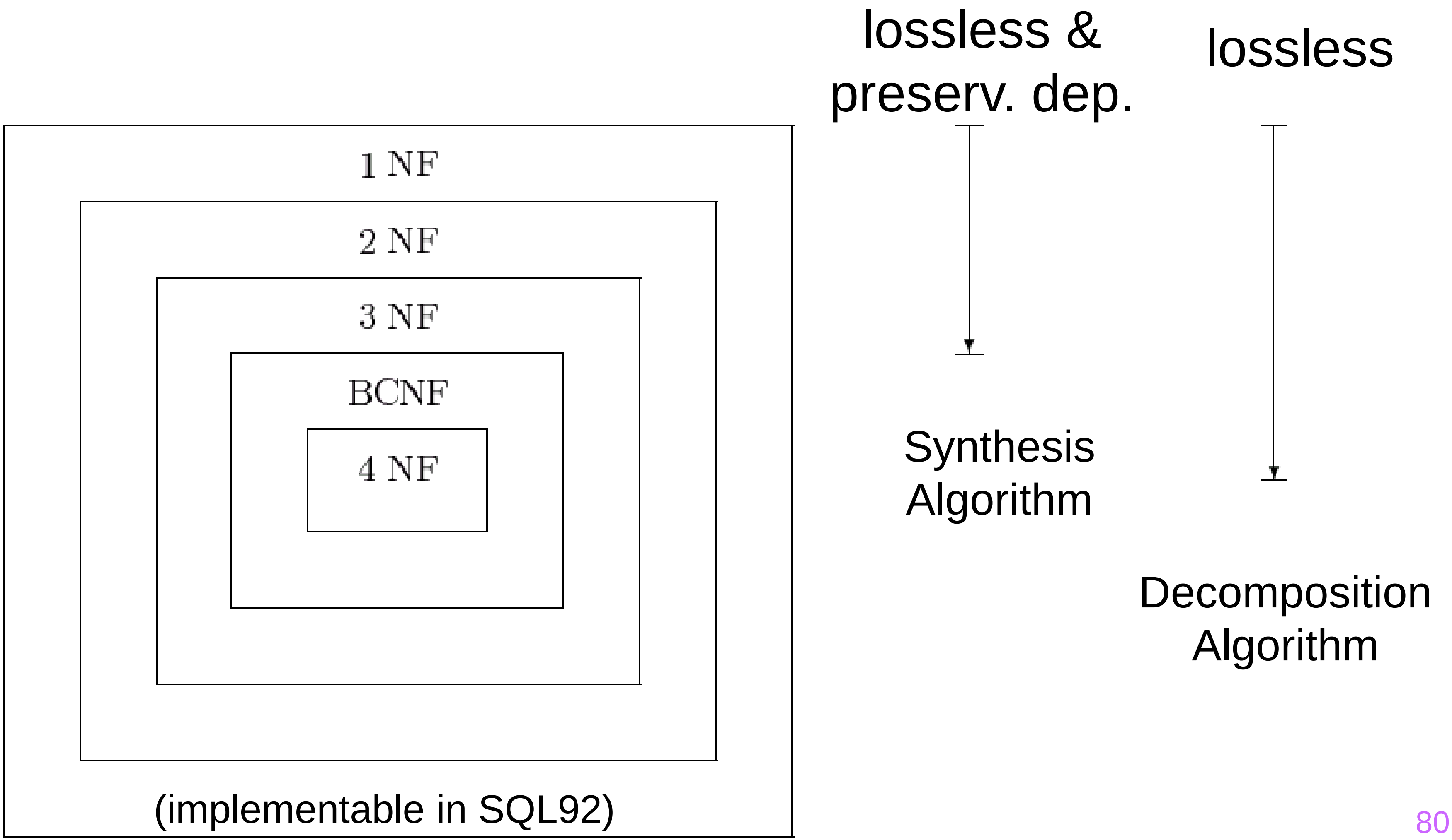
- Assistant: {[PersNr, Name, Area, Boss, Language, Progr.]}

- Synthesis Algorithm (3NF)
  - Assistant: {[PersNr, Name, Area, Boss]}
  - Skills: {[PersNr, Language, Programming]}

- Decomposition Algorithm (4NF)
  - Assistant: {[PersNr, Name, Area, Boss]}
  - Languages: {[PersNr, Language]}
  - Programming: {[PersNr, Programming]}

Summary

- Lossless decomposition up to 4NF
- Preserving dependencies up to 3NF

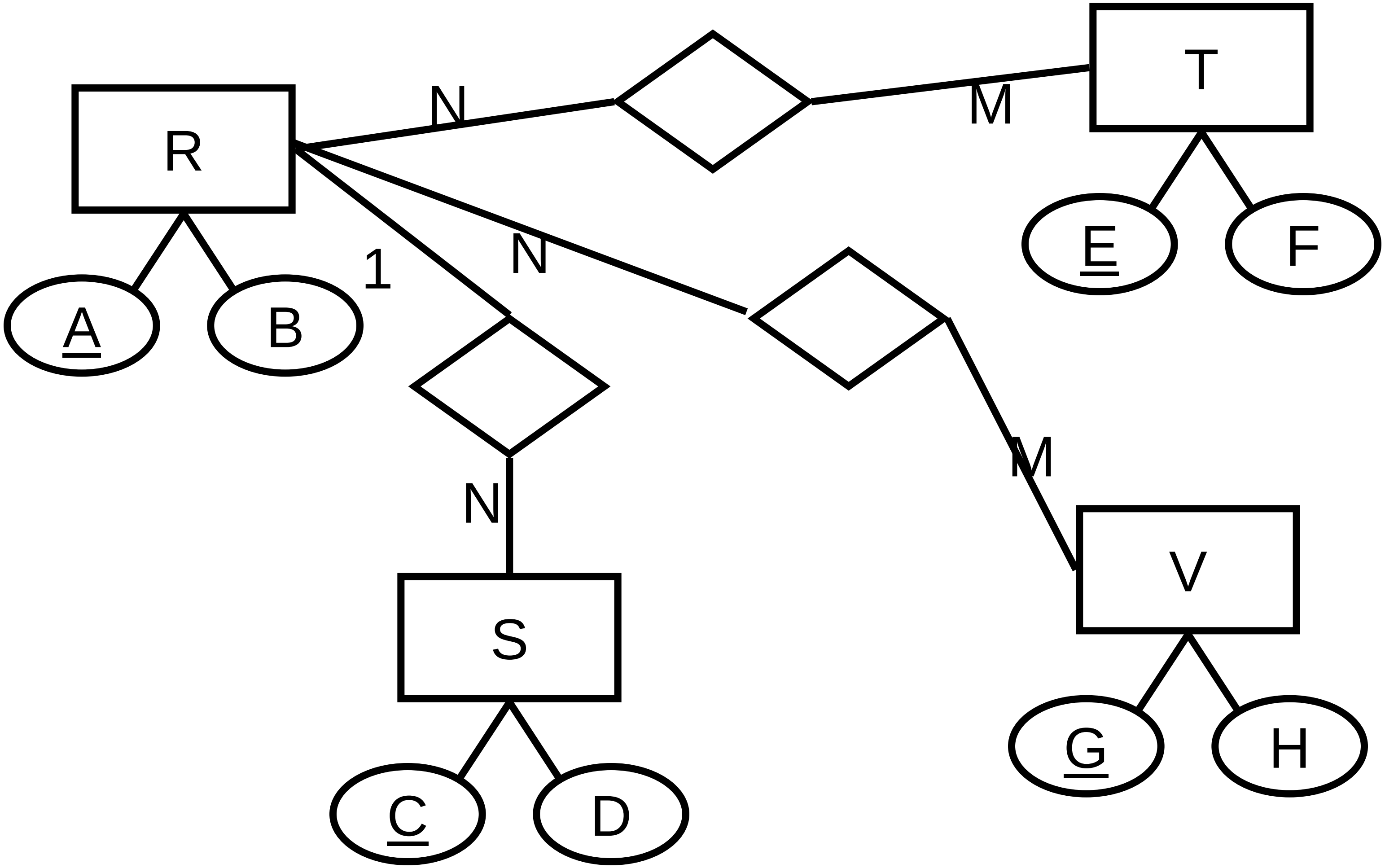


Exercise

Family Tree				
Child	Father	Mother	Grandpa	Grandma
Sofie	**Alfons**	**Sabine**	**Lothar**	**Linde**
Sofie	**Alfons**	**Sabine**	**Hubert**	**Lisa**
Niklas	**Alfons**	**Sabine**	**Lothar**	**Linde**
Niklas	**Alfons**	**Sabine**	**Hubert**	**Lisa**
Tobias	Leo	Bertha	Hubert	Martha
...	...	...	...	...

- Find FDs, MVDs, keys
- Decompose into 3NF, BCNF, 4NF

Exercise: 1:N & N:M Relationships



UR: {[A , B , C , D , E , F , G , H]}