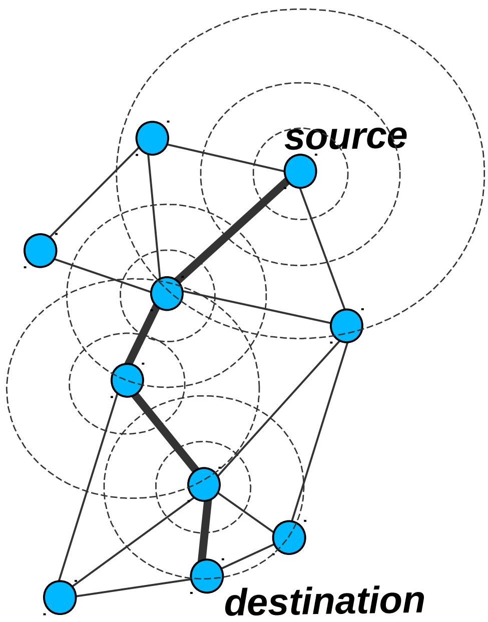
Topology Control

Topology Control

- Sparse topologies, low node degree
  - Storage complexity, storage efficiency
- Short paths, low energy paths
  - Energy: battery life time
    health issues
    (high frequency radiation)
- Low load
- Efficient distributed construction and maintenance
  scalability
  fault tolerance
  self-reconstruction

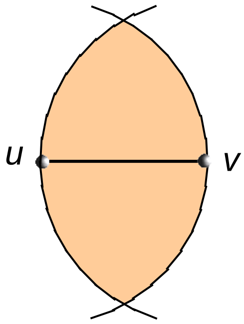


Unit Disk Graph

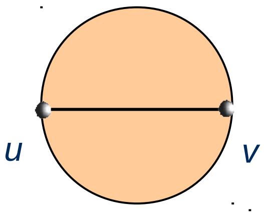
- Let V be a set of n wireless nodes in a 2D Euclidean plane
- These nodes define a unit disk graph UDG(V)
- ‖uv‖ is the Euclidean distance between nodes u and v
- There is an edge between nodes u and v iff ‖uv‖ <= 1

Planar topologies

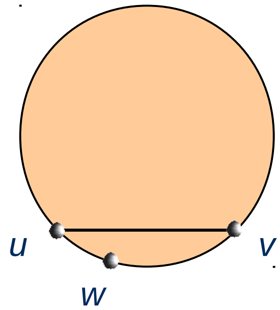
- Relative neighborhood graph *RNG(V)*
  $E_{RNG} = \{(u,v) : \forall\, w \in V,\ u \neq w \neq v,\ ||u,w|| \geq ||u,v||$ és $||v,w|| \geq ||u,v||\}$



- Gabriel graph *GG(V)*
  $E_{GG} = \{(u,v) : \forall\, w \in V,\ u \neq w \neq v,\ w \notin D(u,v)\}$,
  where D(u,v) is the (interior of the) disk with diameter uv

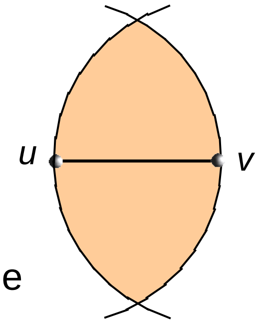


- Delaunay triangulation *Del(V)*
  $E_{Del} = \{(u,v) : \exists\, w \in V,\ u \neq w \neq v,\ \forall\, w' \in V,\ w' \notin D(u,v,w)\}$,
  where D(u,v,w) is the (interior of the) disk, which contains u,v,w on the boundary

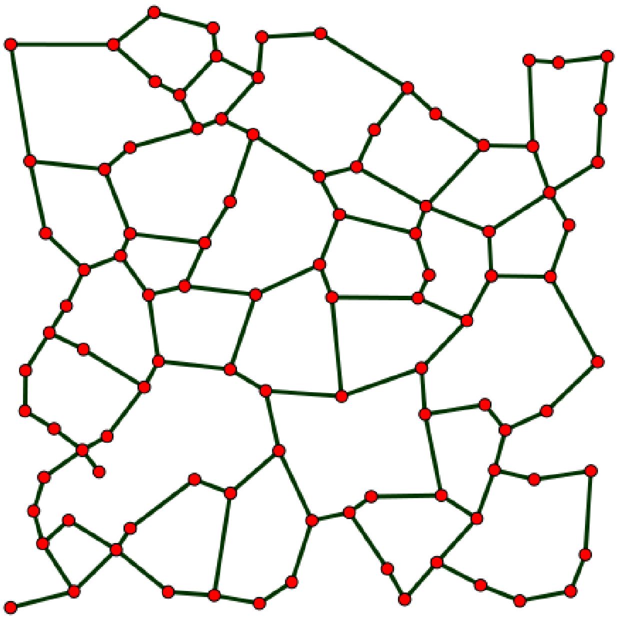


- $RNG(V) \subseteq GG(V) \subseteq Del(V)$

Relative Neighborhood Graph

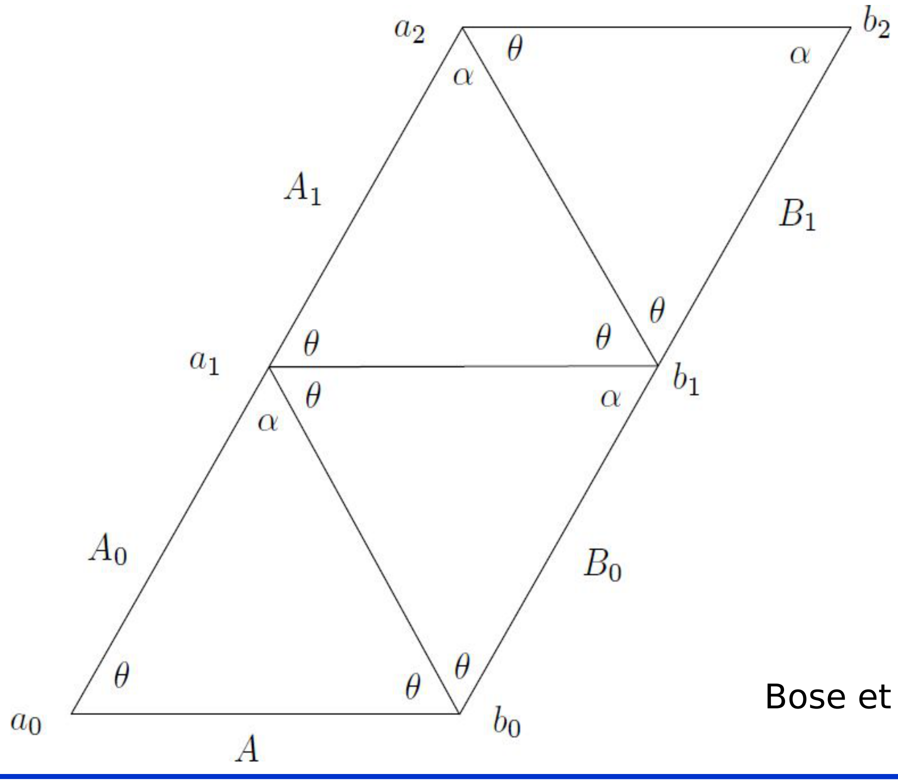


- $E_{RNG} = \{(u,v) : \forall\, w \in V,\ u \neq w \neq v,\ \|u,w\| \geq \|u,v\|$ és $\|v,w\| \geq \|u,v\|\}$



- Locally computable
- Connected
- Contains the Euclidean minimum spanning tree [Toussaint 1980]
- Spanning ratio can be as high as $\Omega(n)$ [Bose et al. 2006]
- Power spanner ratio can be as high as n-1 [Li et al. 2001]

Spanning Ratio of the RNG

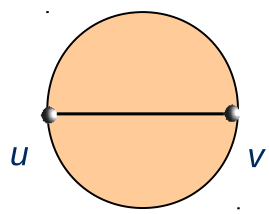


Bose et al. 2006

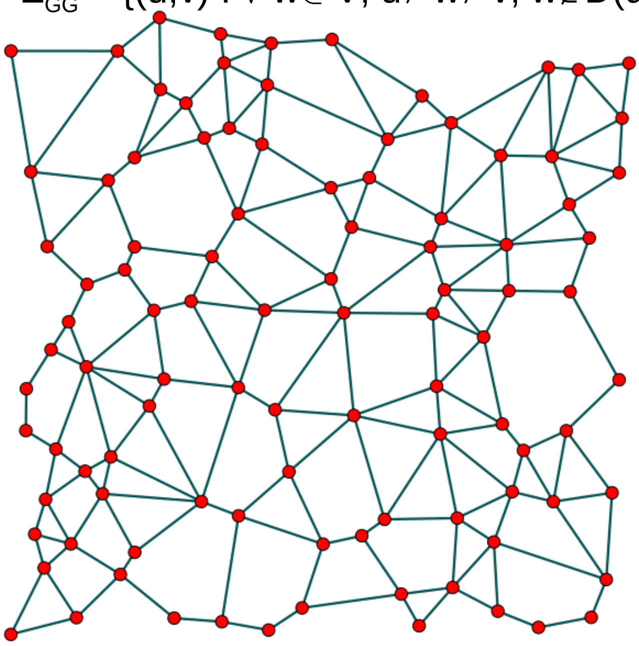
Power Spanning ratio of the RNG

- At most n – 1
- Path between u and v in EMST(V) has at most n – 1 edges and each edge has length at most ‖uv‖
- EMST(V) ⊂ RNG(V) if UDG(V) is connected

Gabriel Graph



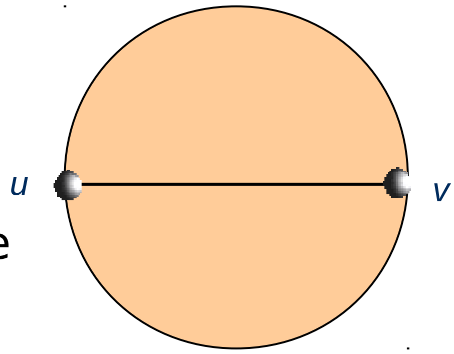
- $E_{GG} = \{(u,v) : \forall\, w \in V,\ u \neq w \neq v,\ w \notin D(u,v)\}$,



- Locally computable
- Contains the RNG
- Spanning ratio is $\Theta(n^{1/2})$ [Bose et al. 2006]
- Power spanning ratio is 1 [Li et al. 2001]

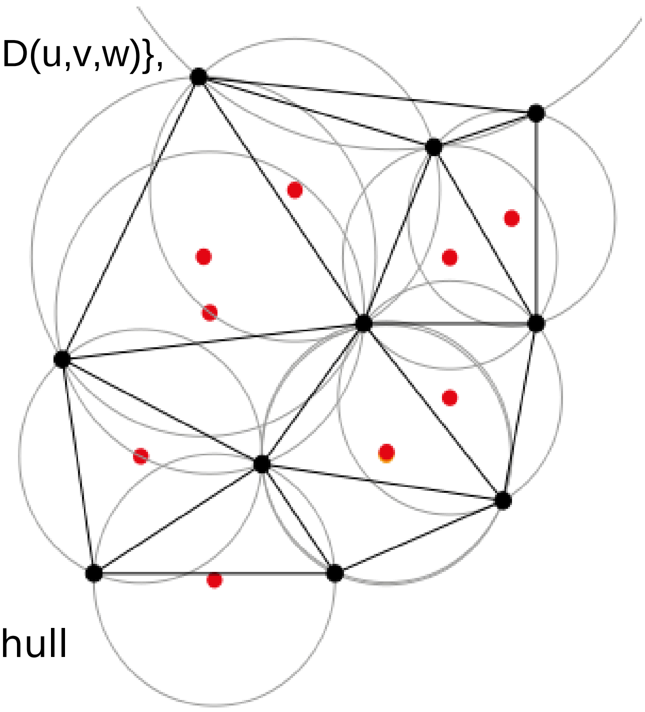
Gabriel Graph – Power Spanning Ratio

- Power spanning ratio is always = 1; Gabriel Graph is an 1-power spanner
- Proof involves showing that no edge can be added to the GG which reduces the energy
- Degree may be Ω(n)



Delaunay Graph

- $E_{Del} = \{(u,v) : \exists\, w \in V,\ u \neq w \neq v,\ \forall\, w' \in V,\ w' \notin D(u,v,w)\}$, where D(u,v,w) is the (interior of the) disk, which contains u,v,w on the boundary
  - *RNG(V) ⊆ GG(V) ⊆ Del(V)*
  - In the plane, each vertex has on average six surrounding triangles.
  - In the plane, the Delaunay triangulation maximizes the minimum angle among all triangulations.
  - Contains the edges of the convex hull

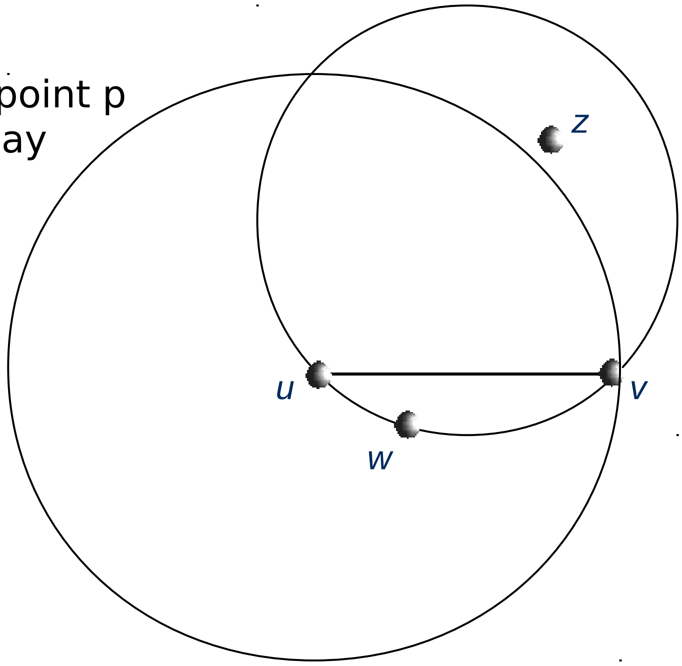


Delaunay Graph

- The closest neighbor b to any point p is on an edge bp in the Delaunay triangulation
- Spanner with spanning ratio

  $$\frac{4\pi}{3\sqrt{3}} \approx 2.418$$

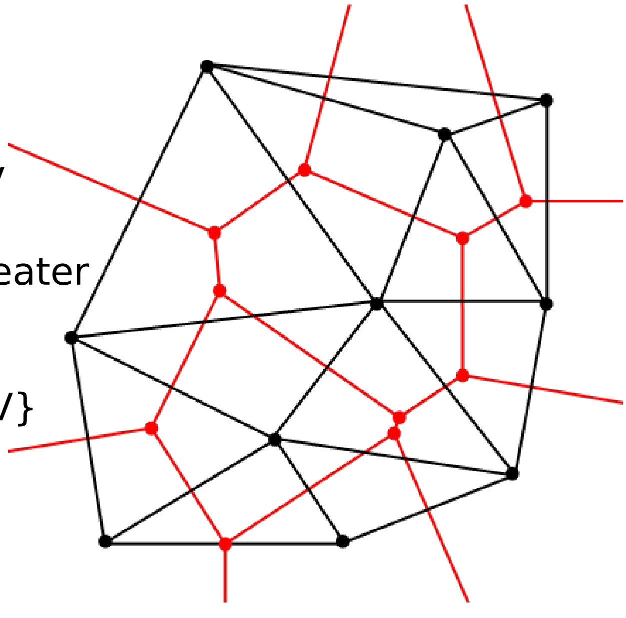
  [Keil, Gutwin 1992]
- Degree can be as high as $\Omega(n)$
- Not computable locally



Delaunay Triangulation – Dual Graph: Voronoi Diagram

- The panar dual of the Delaunay Trianulation DT(V) of the set of points V is called the Voronoi Diagram VD(V) of V
- The Voronoi cell R(p) of a point p∈V is the set of points in the plane the distance of which to p is not greater than to any other point of V

  $R(p)=\{x\in R^2 : ||x,p|| <= ||x,q||, q\in V\}$

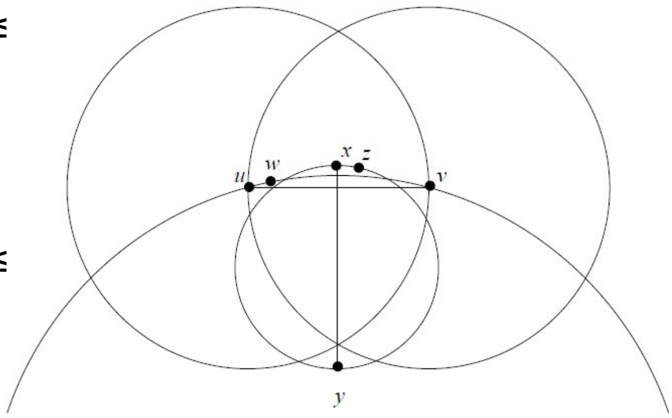


K-Localized Delaunay Graph LDel^k

- A triangle uvw satisfies k-localized Delaunay property if
  - the interior of the circumcircle D(u,v,w) does not contain any node of V that is a k-neighbor of u, v, or w;
  - and all edges of the triangle uvw have length <= 1unit.
  - Triangle uvw is called a k-localized Delaunay triangle
- The k-localized Delaunay graph over a node set V , denoted by LDel^k(V), has exactly all Gabriel edges and the edges of all k-localized Delaunay triangles

K-Localized Delaunay Graph LDel^k

- Theorem [Li et al. 2003]:
  - LDel^k(V) contains the edges of DT(V), $1 \le k \le n$.
  - LDel^k(V) is a spanner, $1 \le k \le$
  - LDel^{k+1}(V) $\subseteq$ LDel^k(V), $1 \le k$
  - LDel^1(V) may be non-planar.
  - LDel^k(V) is planar, for $2 \le k \le$



LDel^1(V) may be non-planar

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