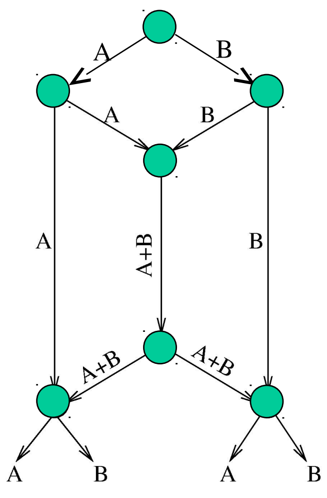
Ad Hoc Networks

Network Coding

Network Coding

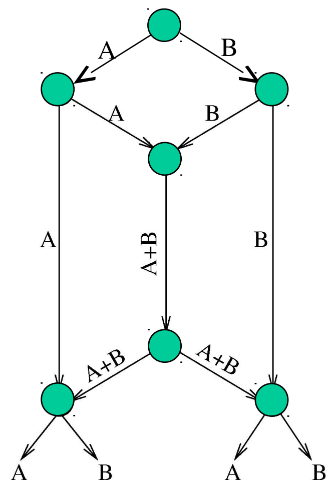
- R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", *IEEE Transactions on Information Theory*, IT-46, pp. 1204-1216, 2000

- The nodes of the network are able to combine the received packets.
  They send the combined packets through the links
- From a source node to a set of destination nodes the rate of the multicast can achieve the „cut bound" by using network coding
- Generally, it is not possible if nodes can only forward and replicate the pakets



Network Coding and Flows

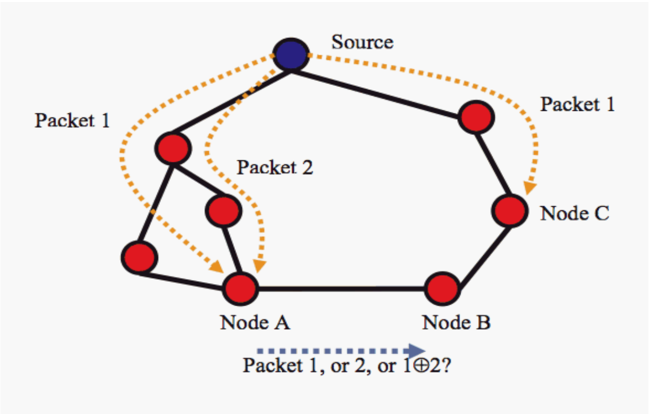
- R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network Information Flow", *IEEE Transactions on Information Theory*, IT-46, pp. 1204-1216, 2000
- Theorem [Ahlswede et al.]
  - There is a network code for each graph such that each target nodes receives as much information as the maximal flow problem for each target allows



Practical Network Coding in Peer-to-Peer Networks

- Christos Gkantsidis, Pablo Rodriguez: Network coding for large scale content distribution. Proc. INFOCOM, pp. 2235-2245, 2005.

- Goal
  - Overcome the „coupon collector problem" in the data distribution
    - A message of m blocks can be received if the number of received encoded blocks is at least m
  - Optimal transmission of files with respect to the available bandwidth



Practical Network Coding in Peer-to-Peer Networks

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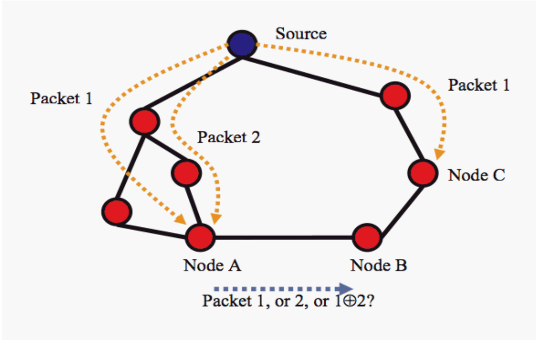
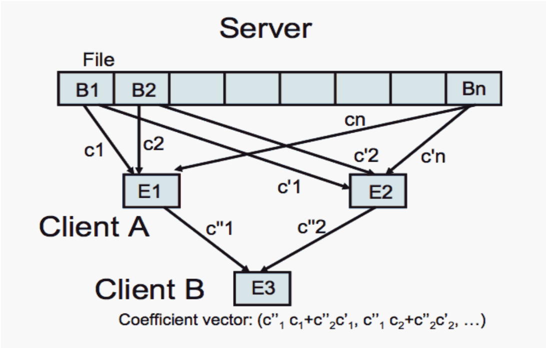
- Method
  - Use linear combinations of the blocks of the message
    - Send combination with the corresponding coding coefficients
  - Recombine transmitted encoded blocks in intermediate stations
  - Receivers collect the linar combinations
  - Use matrix inverse defined by the coding coefficients to reconstruct the original blocks





Encoding and Decoding

- Original message blocks: $x_1, x_2, \ldots, x_m$
- Random variables r_{ij}
- Encoded blocks: $y_1, y_2, \ldots, y_m$

$$(r_{i1} r_{i2} \ldots r_{im}) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = y_i$$

- Hence

$$\begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

Encoding and Decoding

- Encoding:

$$\begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

- Decoding: If the matrix (r_{ij}) is invertable, then we have

$$\begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} r_{11} & \cdots & r_{1m} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mm} \end{pmatrix}^{-1} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$$

Inverting a Random Matrix

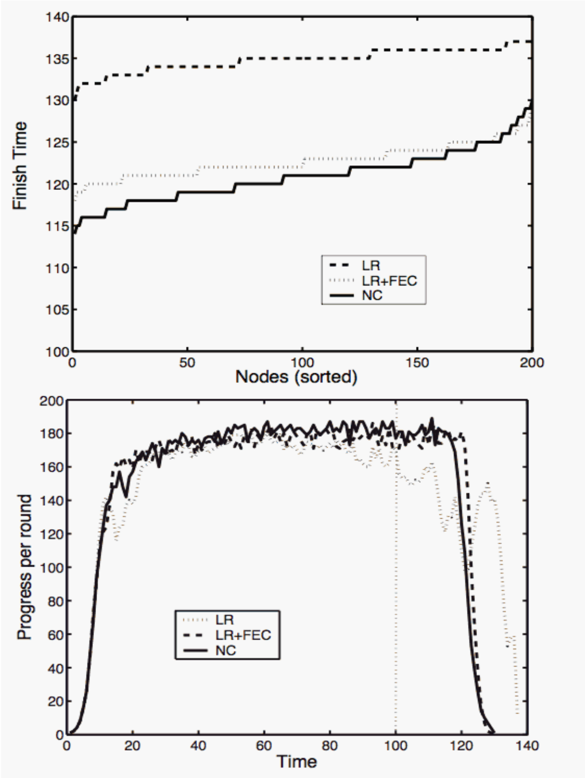
- If the numbers of a m x m random matrix are chosen uniformly and independently from a finite field of size b, then the random matrix can be inverted with probability of at least:

$$1 - \sum_{i=1}^{m} \frac{1}{b^i}$$

- Idea: Choose finite field $GF[2^8]$
  - Computation with bytes is very efficient
  - The success probability is at least 0.99
  - In case of error an additional frame gives again a success probability of at least 0.99

Speed of Network Coding in Peer-to-Peer-Networks

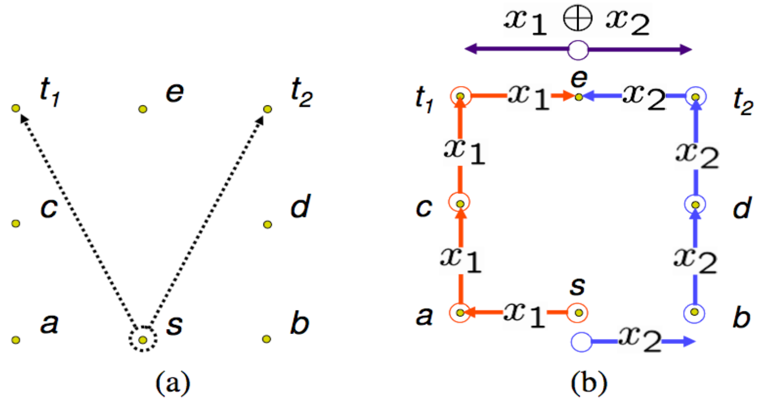
- Comparison
  - Network-Coding (NC) versus
  - Local-Rarest (LR) and
  - Local-Rarest+Forward-Error-Correction (LR+FEC)



Minimum-Energy Multicast in Ad Hoc Networks

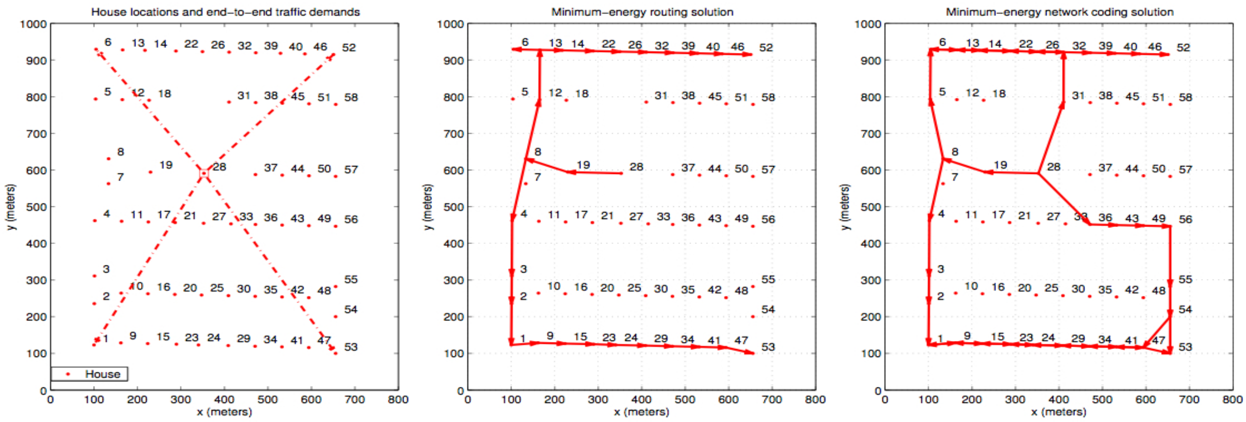
- Yunnan Wu, Philip A. Chou, Sun-Yuan Kung: Minimum-energy multicast in mobile ad hoc networks using network coding. IEEE Transactions on Communications 53(11): 1906-1918, 2005.

- Multicast: Send message from one node to a dedicated set of nodes
- Example:
  - Traditional cost: 5 energy units for 1 message
  - With network coding: 9 energy units for 2 messages



Minimum-Energy Multicast in Ad Hoc Networks

- Yunnan Wu, Philip A. Chou, Sun-Yuan Kung: Minimum-energy multicast in mobile ad hoc networks using network coding. IEEE Transactions on Communications 53(11): 1906-1918, 2005.
- Solving minimal energy multicasting is NP-hard
  - Problem: Solve an integer linear optimization problem
- With network coding the maximum throughput can be found in polynomial time

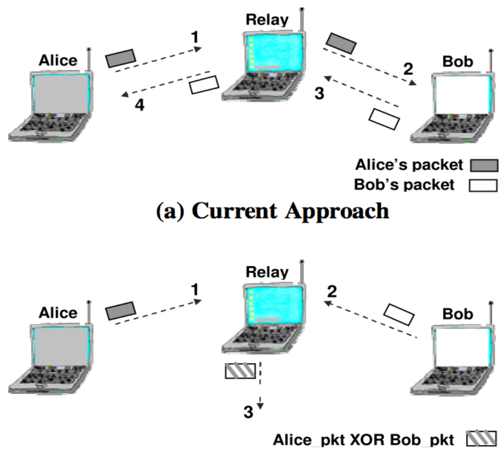


(a) (b) (c)

XORs in the Air

- Sachin Katti, Hariharan Rahul, Wenjun Hu, Dina Katabi, Muriel Médard, Jon Crowcroft: XORs in the air: practical wireless network coding. Proc. SIGCOMM, 243-254, 2006.

- Problem:
  - Maximize throughput in an ad hoc network
  - Multihop messages lead to interferences
- Example
  - Traditional: 4 messages to deliver a message from Alice to Bob and from B
  - Network Coding: 3 messages



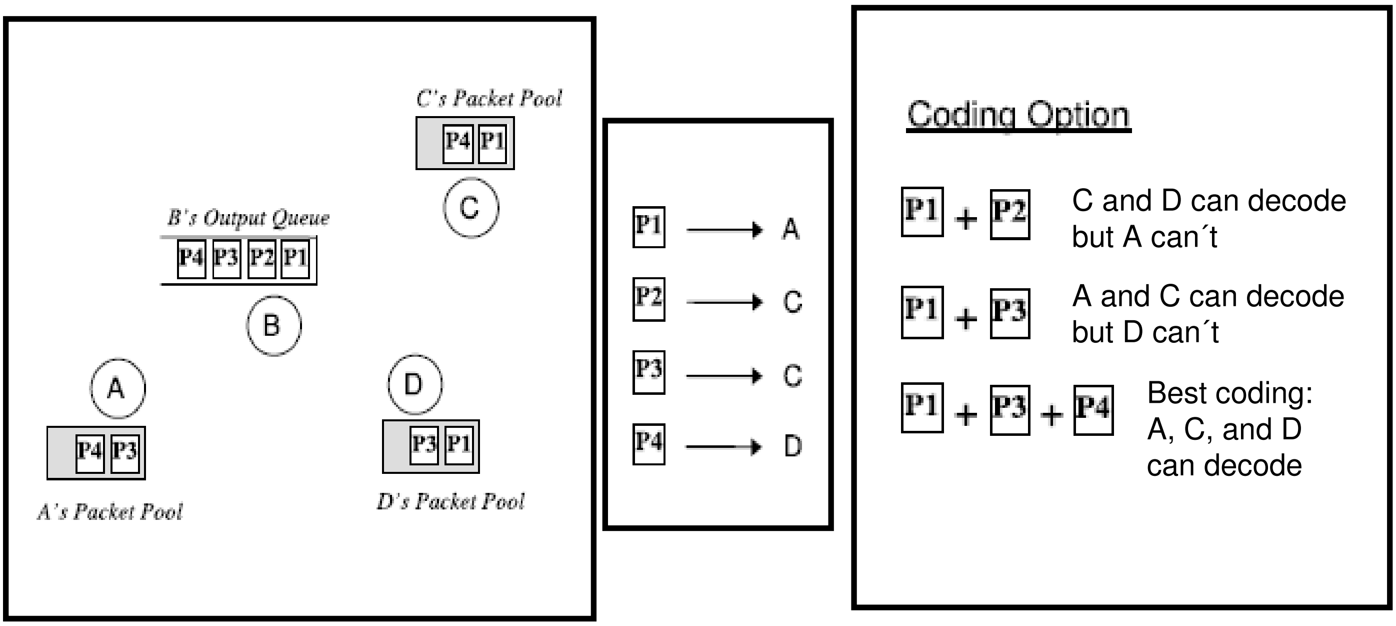
(a) Current Approach

(b) COPE

Components of COPE

- Opportunistic Listening
  - Get maximum context for decoding messages
- Opportunistic Coding
  - „The key question is what packets to code together to maximize throughput. A node may have multiple options, but it should aim to maximize the number of native packets delivered in a single transmission, while ensuring that each intended nexthop has enough information to decode its native packet."
- Learning Neighbor State
  - Each node announces the packets it has received
  - Each node also guesses the packets a neighbor could have received

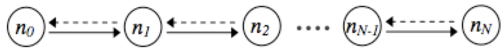
Opportunistic Coding



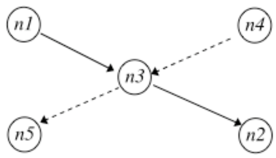
B can code packets it wants to send

Coding Gain, MAC Gain

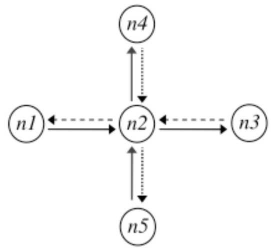
- Coding Gain:
  - Number of messages saved because of network coding
- Coding+MAC Gain:
  - Intermediate routers forming a bottleneck further delay the medium access
  - Using COPE an additional speedup occurs



Chain topology: 2 flows in reverse directions.



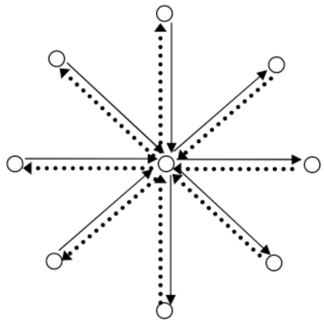
X- topology: 2 flows intersecting at n_3.



Cross topology: 4 flows intersecting at n_2.

Topology	Coding Gain	Coding+MAC Gain
Alice-and-Bob	1.33	2
"X"	1.33	2
Cross	1.6	4
Infinite Chain	2	2
Infinite Wheel	2	∞

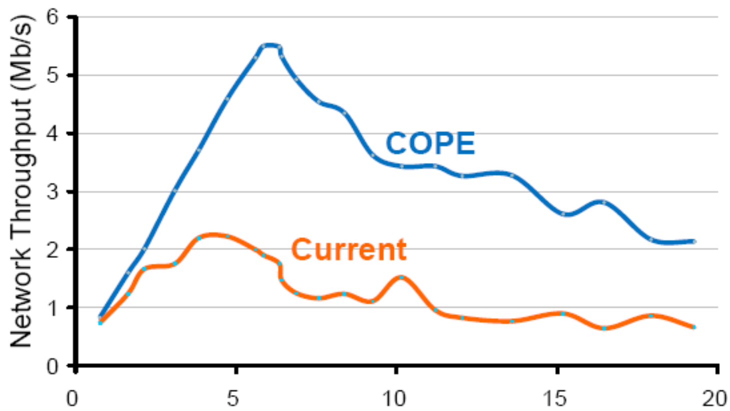
Theoretical gains for a few topologies.



Wheel topology: many flows intersecting at the center node

Summary

- Network Coding can help to
  - increase traffic throughput in Ad Hoc Networks
  - decrease energy consumption in multicast
  - increase robustness and reduce the error rate
  - increase throughput in Peer-to-Peer Networks
  - increase throughput in Wireless Sensor Networks
- Network Coding schemes suffer from the computation costs of inverting large matrices and introduce a delay for decoding.
- COPE is an exception: efficient and does not cause delay.



Cope can provide a several-fold (3-4) increase in the throughput of wireless ad hoc networks. Results are for UDP flows with randomly picked source-destination pairs, Poisson arrivals, and heavy-tail size distribution.

Literature

- R. Ahlswede, N. Cai, S.-Y. R. Li, R. W. Yeung: Network Information Flow, *IEEE Transactions on Information Theory*, IT-46, 1204-1216, 2000.
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