

Fast Collisionless Pattern Formation by Anonymous, Position-Aware Robots

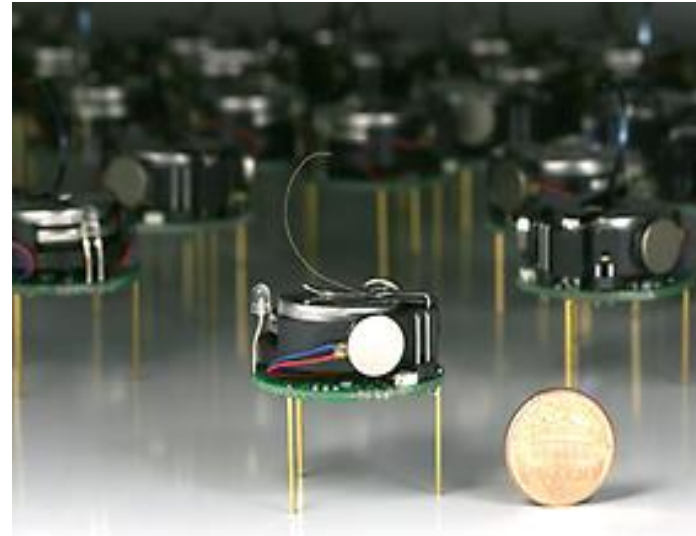
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Pattern Formation

- n robots with restricted capabilities
 - Limited viewing range
 - No communication
- 2D plane setting (2D grid)
- They want to form a given connected pattern
 - Special case:
gather at a given point
(Point formation)



<http://www.eecs.harvard.edu/ssr/projects/progSA/kilobot.html>

Where is it needed?

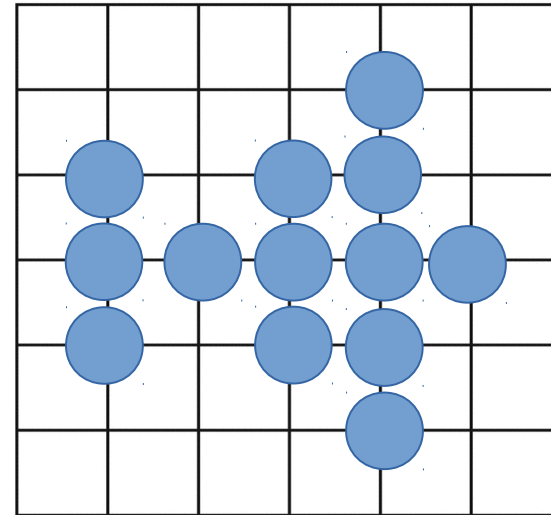
- Gathering:
 - Exploration Robots
 - To exchange data they need to gather
 - Task splitting
 - After gathering the main robot distributes the tasks
 - ...
- Pattern formation
 - Flight array
 - Self-deploying mobile sensors
 - Self-organizing particle system

Overview

- Model
- Problems
- Related work
- Our contribution
- Summary

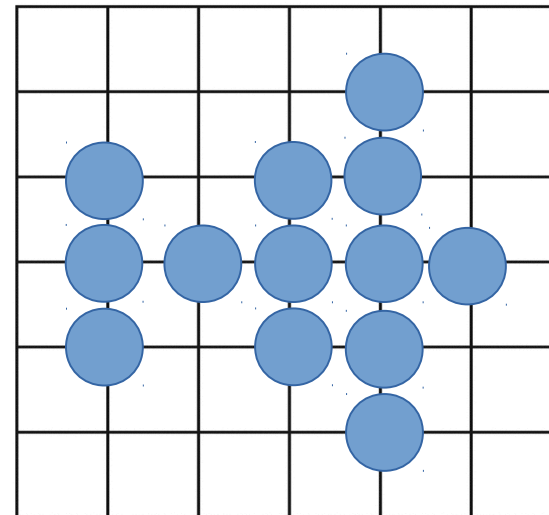
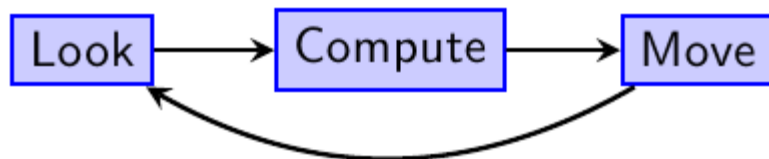
Anonymous Restricted Robots

- Identical and anonymous
- Oblivious (for gathering)
 - finite state, i.e. $O(1)$ bits of persistent memory for pattern formation
- No communication
- Limited sight: 2 units
- Represented by disk of unit diameter
- Aware of own position in Z^2
- Common coordinate system
- Can move on the edges of the grid



Synchronous Look-Compute-Move (LCM)

- In each time unit each robot can move to a neighboring grid vertex or stay idle
- Collision is not allowed
- Synchronous Look-Compute-Move cycles

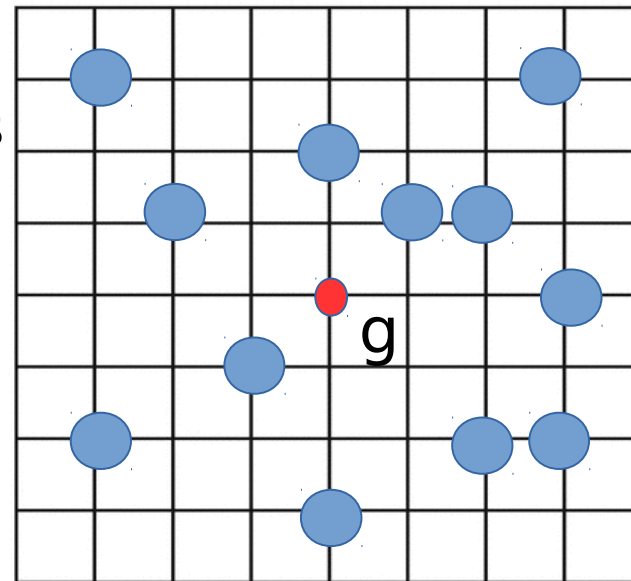


- **Look:** take snapshot of its 2-hop neighborhood
- **Compute:** decision to move or stay in place
- **Move:** moving or staying in place

Problems considered

- **Collisionless Gathering**

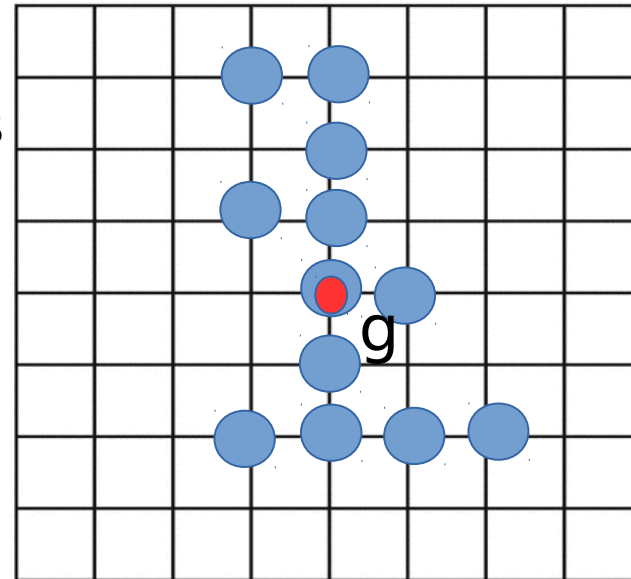
- Given:
 - n robots on different grid vertices
 - gathering vertex g , known for all robots
- Goal:
 - Form a connected configuration containig g
 - Collision is not allowed



Problems considered

- **Collisionless Gathering**

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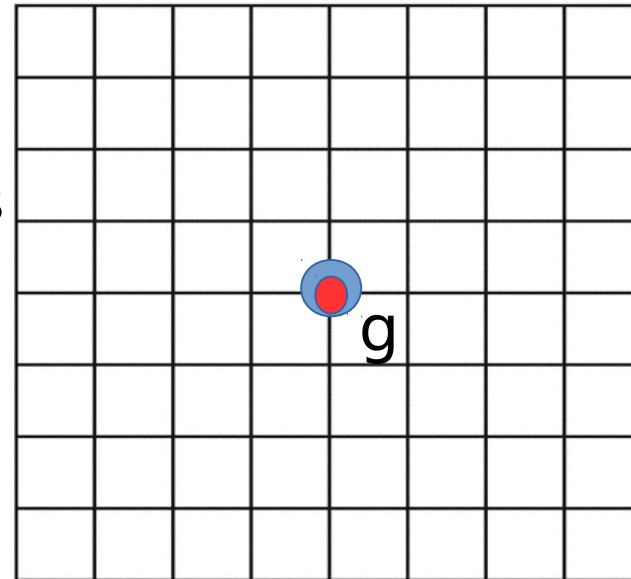


Problems considered

- Helpful intermediate gathering problem:

Lemmings Problem

- Given:
 - n robots on different grid vertices
 - vertex g , known for all robots
- Goal:
 - All robots must occupy g
 - Collision is only allowed at the single vertex g



Problems considered

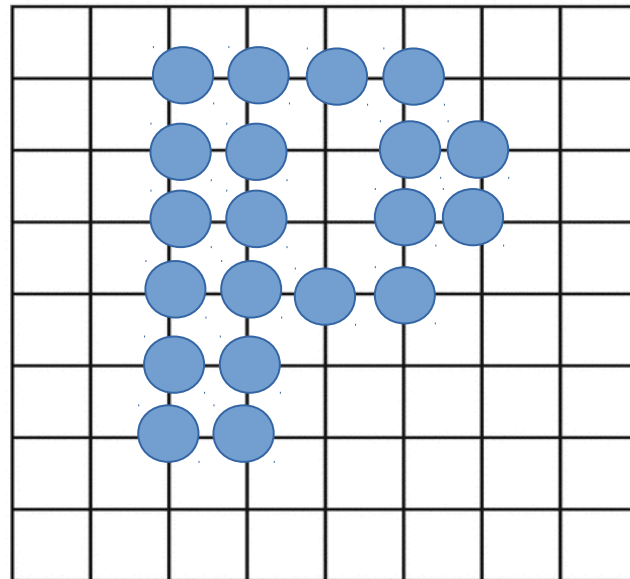
- **Formation of a connected pattern F**

- Given:

- n robots on different grid vertices
- pattern P, known for all robots
 - as set of vertices, or
 - partially described, e.g. build a connected pattern with minimum diameter

- Goal:

- Form P
- Collision is not allowed

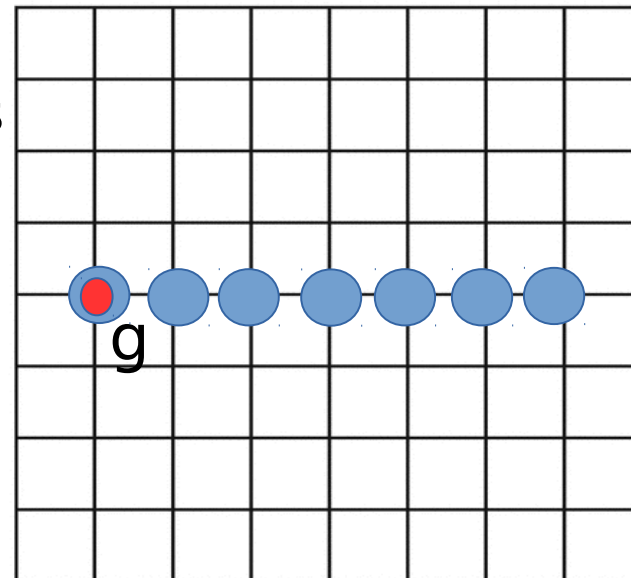


Problems considered

Special case of pattern formation:

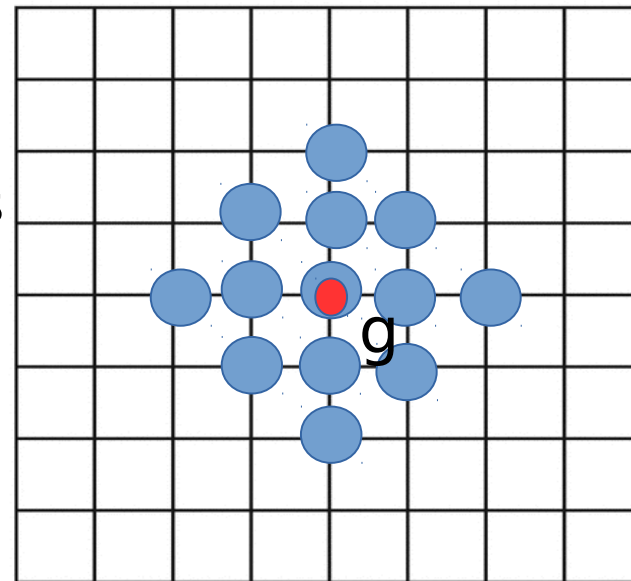
Axis parallel line segment formation

- Given:
 - n robots on different grid vertices
 - vertex g known for all robots
- Goal:
 - Form an axis parallel line segment with end point g



Problems considered

- Special case of pattern formation:
Focused coverage
 - Given:
 - n robots on different grid vertices
 - vertex g (Point-of-Interest, POI), known for all robots
 - Goal:
 - Cover all vertices of an area with maximum radius around the POI without holes
 - Collision is not allowed



Related work

- Survey on gathering by oblivious robots [Cileiebak et al. SICOMP 2012]
- Asynchronous gathering of oblivious point-like robots with infinite visibility in LCM model in finite time [Cohen Peleg, SICOMP 2005]
- Gathering of fat robots with infinite visibility in finite time [Czyzowicz et al., TCS 2009]
- Gathering of fat robots on the grid with limited visibility at a given point in time $O(nD)$ time [Chord-Landwehr et al. 2009]
- Formation of geometric patterns by anonymous robots with infinite visibility [Suzuki, Yamashita, SICOMP 1999]
- Pattern formation by robots with limited visibility [Suzuki, Yamashita, SIROCCO 2013]
- Focused coverage by self deploying mobile sensors in finite time [Li et al. TMC 2011]
- Focused coverage in $O(nD)$ time [Blazovics, Lukovszki ALGOENORS 2013]

Our Contribution (1)

- **Lemmings Problem:**

- Algorithm for oblivious robots with visibility range of 2 units solving the problem in $2n + D - 1$ time steps,
 - D is the maximum initial distance of a robot from g
- Lower bound: $\Omega(n + D)$
 - Holds also for robots with infinite visibility range
- Optimal up to a constant factor

Our Contribution (2)

- **Collisionless Gathering:**

- Algorithm for oblivious robots with visibility range of 2 units solving the problem in $n + D - 1$ time steps
- Improves previous upper bound of $O(nD)$ [Chord-Landwehr et al. 2011]

Our Contribution (3)

- **Formation of an axis parallel line segment**
 - Algorithm for finite state robots solving the problem in
 $3n + D + 3$ steps
 - Only 2 bits of persistent memory

Our Contribution (4)

- **Formation of a connected pattern F:**
 - Algorithm for finite state robots solving the problem in
 $O(n + D^*)$ time steps,
where D^* is the diameter of the initial scene
 - consisting of F and the initial positions of the robots

Our Contribution (5)

- **Focused Coverage:**

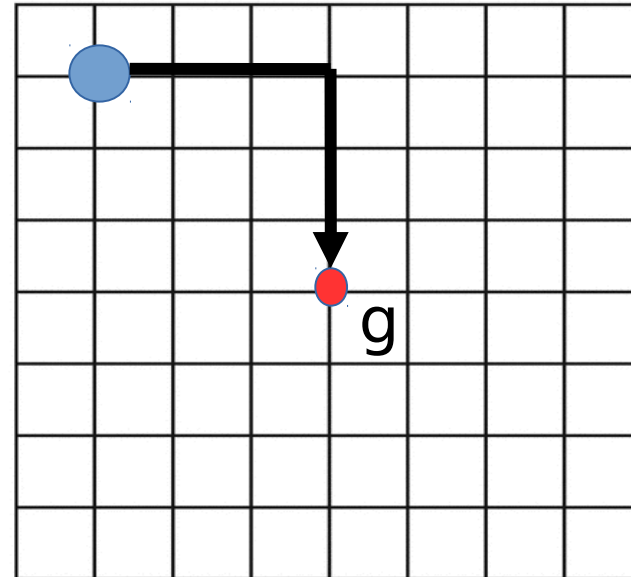
- Algorithm for finite state robots solving the problem in

$O(n + D)$ time steps

- Improves previous upper bound of $O(S)$, where S is the sum of initial distances of the nodes from g [Blazovics, Lukovszki, 2013]

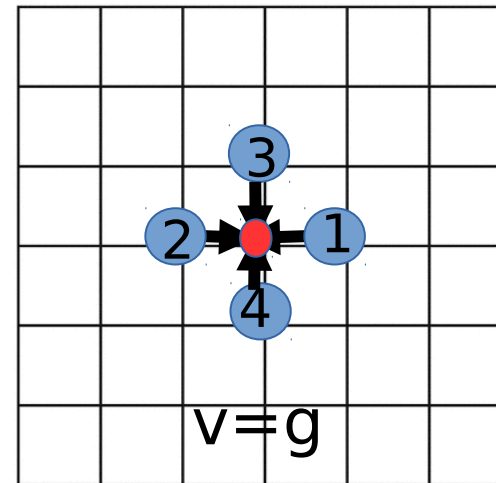
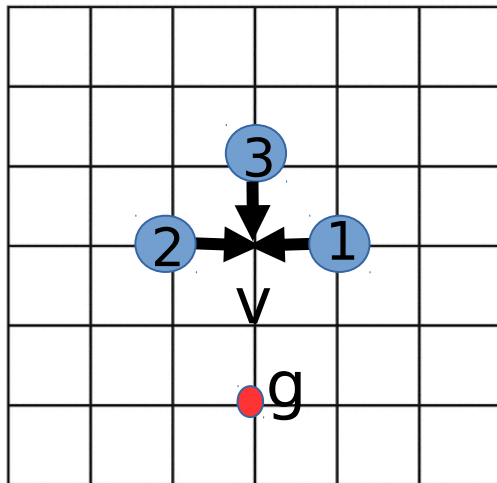
X-Y-Routing

- In each time step each robot wants to decrease its hop distance to g :
 - it moves horizontally until it has the same x-coordinate as g
 - then it moves vertically until it reaches g
- If the next vertex of the x-y-path is occupied, then stay idle



X-Y-Routing

- If 2 or more robots wants to move to the same vertex v :
 - the robot with highest priority moves to v
 - the other robots stay in place



- Collision free
- Each robot knows its 2-hop neighborhood
==> robots can decide locally, which one can move
- Oblivious

X-Y-Routing

Algorithm 1 *x-y-routing*(r)

while r has not yet reached g **do**

$p \leftarrow \text{nexthop}(r, t)$

if p is unoccupied **and** \nexists another robot r' with $\text{nexthop}(r', t) = p$, s.t. r' has higher priority than r **then**

r moves to p

else

r stays in place

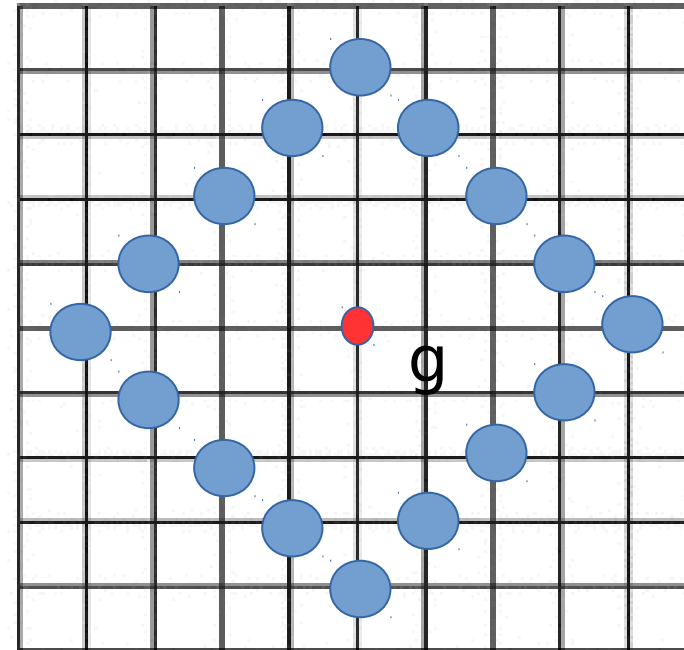
end if

$t \leftarrow t + 1$

end while

Lemmings Problem – Lower Bound

- Each robot must arrive at g
==> at least D steps
- In each time step at most 4 robots
can reach g (1 from each direction)
==> at least $n/4$ steps



- **Theorem 1:** Let R be a set of n robots with infinite visibility placed on different vertices of Z^2 . Each algorithm solving the synchronous lemmings problem needs $\Omega(n + D)$ time steps.

Lemmings Problem – Upper Bound

Perform the x-y-routing algorithm with the modification:
treat g as it would be always an unoccupied one

Theorem 2: Let R be a set of n oblivious robots placed at different vertices of Z^2 . Let g be the gathering vertex. By performing the x-y-routing algorithm, all robots reach g in

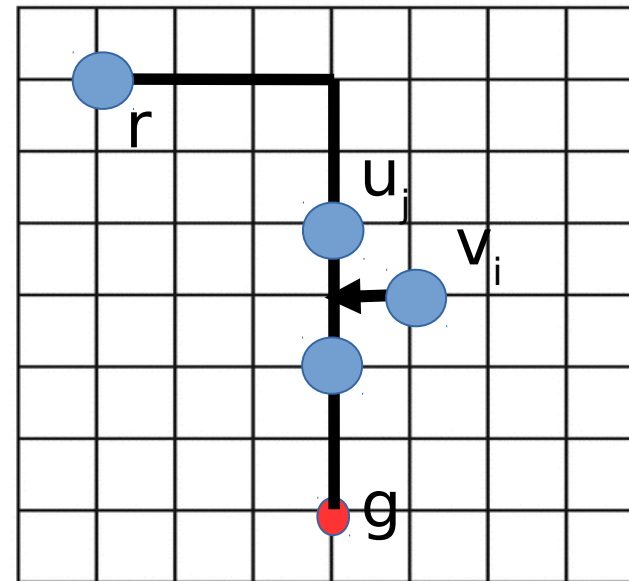
$$2n + D - 1 \text{ time steps,}$$

where D is the maximum initial distance of a robot from g .

Lemmings Problem – Upper Bound

Proof strategy:

- Assume first, all robots are placed on the same x-y-path P terminating at g
- Show by induction: by x-y routing, all robots reach g within $n+D-1$ time
- Consider the robot r arriving at g as the last one
- Let P be the x-y-path from the initial position of r to g
- Let $U = \{u_1, \dots, u_k\}$, $u_k = r$ be the robots on P
- If we remove all robots $R \setminus U$, all robots of U would arrive at g within $k+D'-1$ steps
- Show that each v_i in $R \setminus U$ can increase the arrival time of r by at most 2 time steps



Collisionless Gathering

Perform the (original) x-y-routing algorithm

Theorem 3: Let R be a set of n oblivious robots placed on different vertices of Z^2 . Let g be the gathering vertex. By performing the x-y-routing algorithm, the robots form a connected configuration containing g in

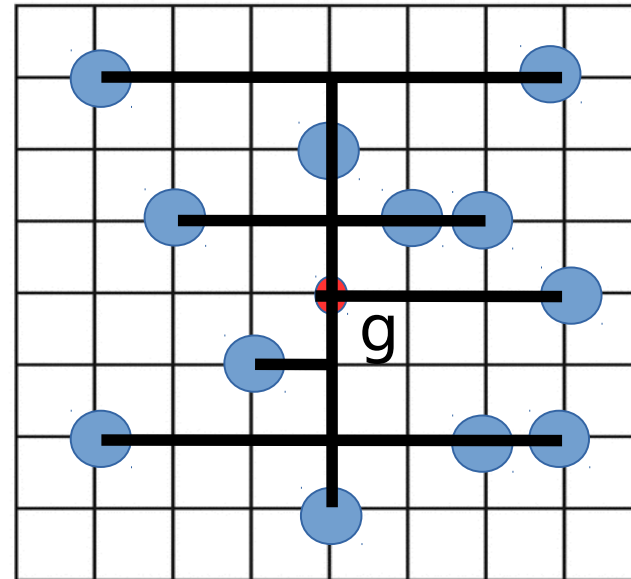
$n + D - 1$ time steps,

where D is the maximum initial distance of a robot from g .

Collisionless Gathering

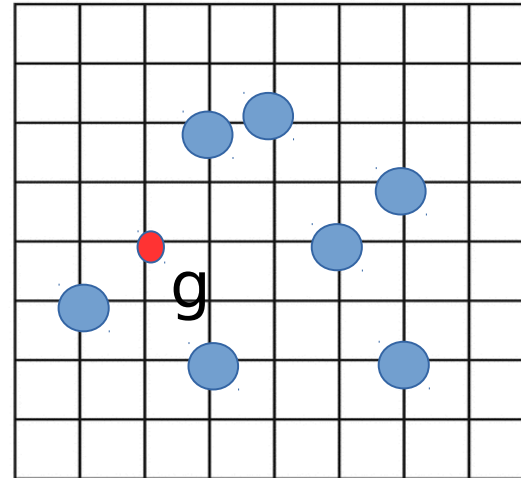
Proof strategy:

- For each robot r , let
 - p_r^* be closest point of r to g during the algorithm,
 - P_r^* be the x-y path from p_r^* to g
- Let $T^* = \bigcup_r P_r^*$
- Show that
 - (i) T^* contains g
 - (ii) T^* has no unoccupied vertex, i. e. T^* is connected
 - (iii) each vertex of T^* becomes occupied in $n+D-1$ steps
 - By induction, prove that after $i+D$ steps all vertices of T^* with hop distance at most i from g are occupied



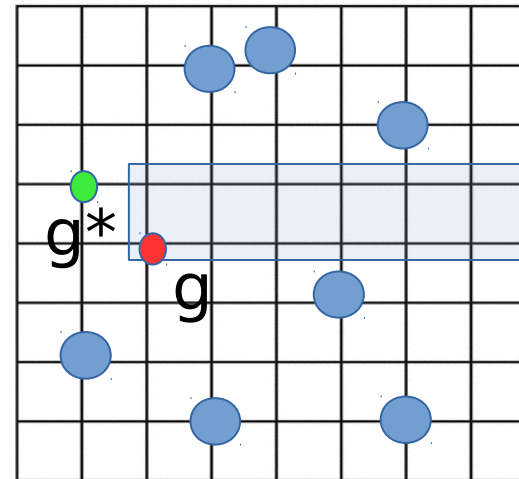
Axis-Parallel Line Segment Formation

- 1) Each robot with y -coordinate $> g_y$
moves one step upwards
Each robot with y -coordinate $\leq g_y$
moves one step downwards



Axis-Parallel Line Segment Formation

- 1) Each robot with y -coordinate $> g_y$ moves one step upwards
Each robot with y -coordinate $\leq g_y$ moves one step downwards
- 2) Let $g^* = (g_x - 1, g_y + 1)$.
Execute the Lemmings algorithm with sink g^* .
When a robot r occupies g^* in step t , r moves one hop to the right in step $t+1$.



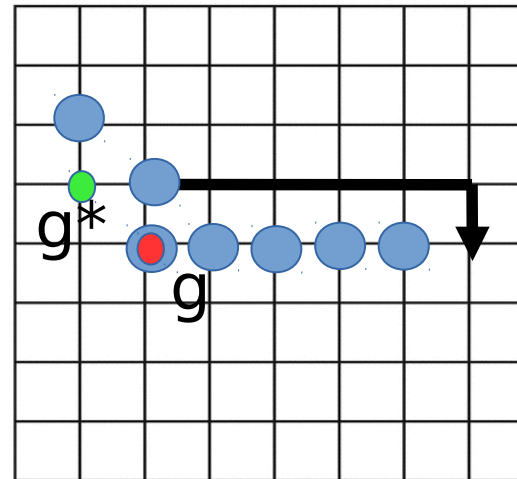
Forming Axis-Parallel Line Segment

- 1) Each robot with y -coordinate $> g_y$ moves one step upwards
Each robot with y -coordinate $\leq g_y$ moves one step downwards
- 2) Let $g^* = (g_x - 1, g_y + 1)$.

Execute the Lemmings algorithm with sink g^* .

When a robot r occupies g^* in step t , r moves one hop to the right in step $t+1$

- 3) Build the line segment L as follows:
Until the vertex v below the current position of a robot is occupied move to the right. Otherwise, occupy v and terminate the algorithm of that robot



Axis-Parallel Line Segment Formation

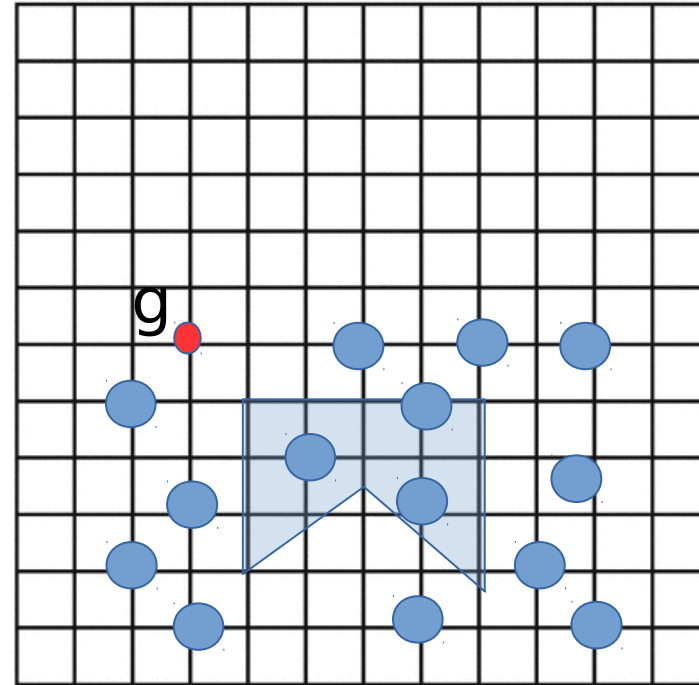
Theorem 4: Let R be a set of n finite state robots placed on different vertices of Z^2 . Let g be a point, known for all robots. Then by the above algorithm the robots form a horizontal line segment with left end point g in

$$3n + D + 3 \text{ time steps,}$$

where D is the maximum initial distance of a robot from g .

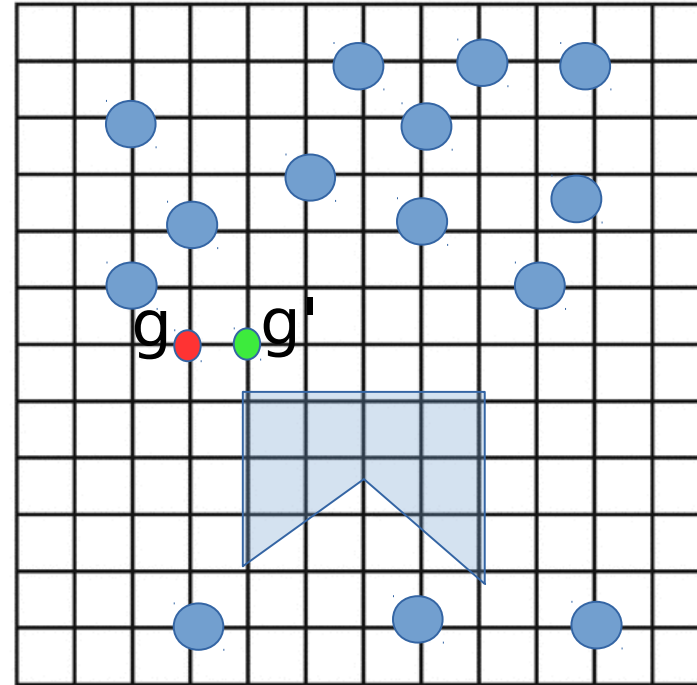
Forming an Arbitrary Connected Pattern F of size n

1) Let y_{\min} and y_{\max} be the minimum and maximum y-coordinate of F. Each robot with y-coordinate $\geq y_{\min}$ moves $y_{\max} - y_{\min} + 2$ steps upwards



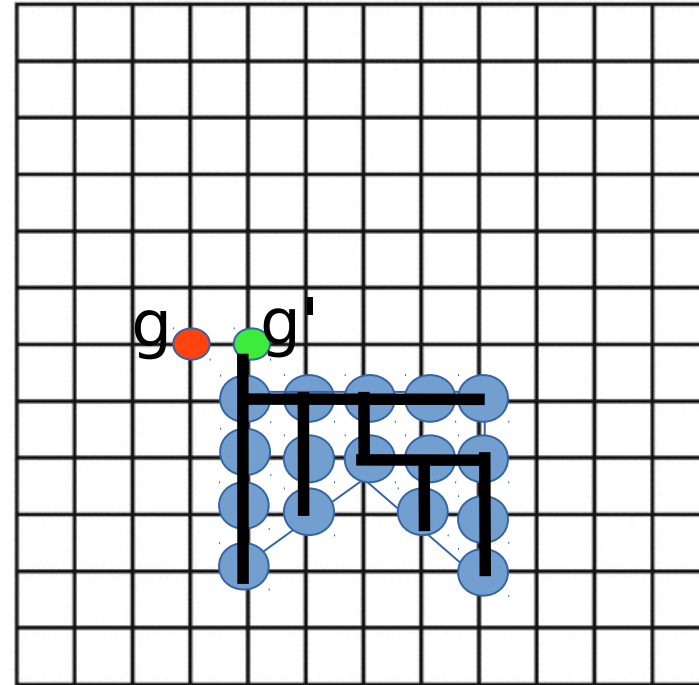
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- 2) Let $g = (g_x - 1, y_{\min} - 1)$, $g' = (g_x, y_{\min} - 1)$. Execute the Lemmings algorithm with sink g. When a robot r occupies g in step t, r moves to g' step t+1



Forming an Arbitrary Connected Pattern F of size n

- 1) Let y_{\min} and y_{\max} be the minimum and maximum y-coordinate of F. Each robot with y-coordinate $\geq y_{\min}$ moves $y_{\max} - y_{\min} + 2$ steps upwards
- 2) Let $g = (g_x - 1, y_{\min} - 1)$, $g' = (g_x, y_{\min} - 1)$. Execute the Lemmings algorithm with sink g . When a robot r occupies g in step t , r moves to g' step $t+1$.
- 3) Let T be a spanning tree of F. Build F from source g' by DFS filling of T , using the arrivals of the robots in g during the Lemmings algorithm as input stream



Forming an Arbitrary Connected Pattern F of size n

Theorem 5: Let R be a set of n finite state robots placed on different vertices of Z^2 . Let F be a connected formation, known for all robots. Then the robots form F in time

$$O(n + D^*),$$

where D^* is the diameter of the point set containing F and the initial positions of the robots.

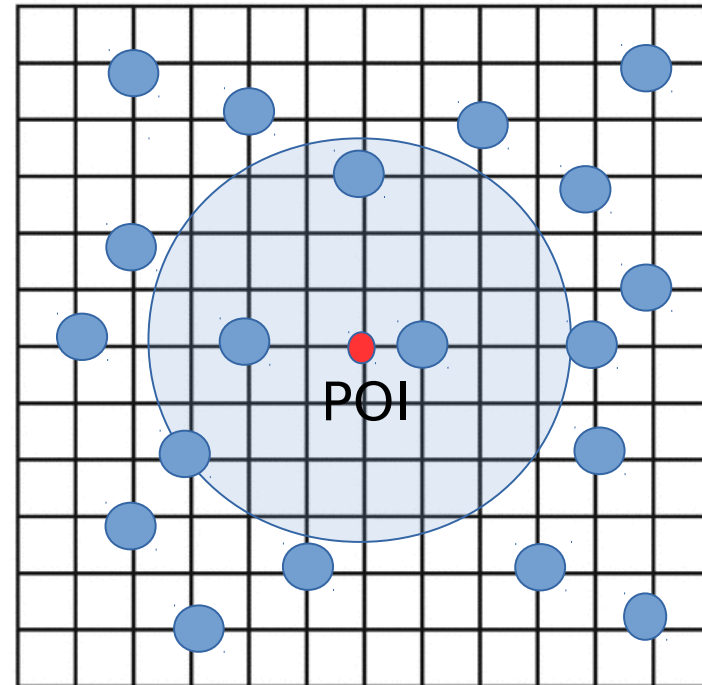
Focused Coverage

- Self deploying mobile sensors
- Solution by oblivious nodes in finite time [Li et al., 2009, 2011]
- $O(nD)$ time [Blazovics, Lukovszki, 2013]

Assume n is known for all nodes

Focused Coverage:

- 1) All robots compute the disc C centered at the POI with maximum radius, which can be covered by n nodes
- 2) Perform the pattern formation algorithm for C



Focused Coverage

Corollary: Let R be a set of n finite state mobile nodes placed on different vertices of Z^2 . Assume n is known for all nodes. Then the nodes form F in time

$$O(n + D),$$

where D is the maximum initial distance of a node from the POI.

Summary

- Introduced the Lemmings problem in the 2D grid
 - helpful tool for Pattern formation
 - Solved by oblivious robots with visibility range of 2 units in **$2n + D - 1$** time step
 - Lower bound of **$\Omega(n + D)$** shown
- Collisionless gathering problem by oblivious robots in the 2D grid in **$n + D - 1$** time steps
 - Improves previous bound of $O(nD)$
- Axis parallel line segment formation by finite state robots in **$3n + D + 3$** time steps
- Formation of an arbitrary connected pattern by finite state robots in **$O(n + D^*)$** time steps
- Focused coverage by finite state nodes in **$O(n+D)$** time
 - Improves previous bound of $O(nD)$

Thank you!