Worst Case Mobility in Ad Hoc Networks

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Abstract

We investigate distributed algorithms for mobile ad hoc networks for moving radio stations with adjustable transmission power in a worst case scenario. We consider two models to find a reasonable restriction on the worst-case mobility. In the pedestrian model we assume a maximum speed $v_{\max}$ of the radio stations, while in the vehicular model we assume a maximum acceleration $a_{\max}$ of the points.

Our goal is to maintain persistent routes with nice communication network properties like hop-distance, energy-consumption, congestion and number of interferences. A route is persistent, if we can guarantee that all edges of this route can be uphold for a given time span Δ, which is a parameter denoting the minimum time the mobile networks needs to adopt changes, i.e. update routing tables, change directory entrees, etc.

We present distributed algorithms based on a grid clustering technique and a high-dimensional representation of the dynamical start situation. We measure the optimality of the output of our algorithm by comparing it with the optimal choice of persistent routes under the same circumstances with respect to pedestrian or vehicular worst-case movements.

1 Introduction

We investigate the problem of constructing a wireless ad hoc network under a worst-case assumption for mobility. For the mobility we consider two different models for the movement of some n mobile stations in the plane, the **velocity bounded** and **acceleration bounded** model.

For the first model we picture to ourselves a large number of **pedestrians** using mobile wireless communication devices in a rather small area. Clearly, the maximum speed is bounded by a small constant. The standard approach in a static ad hoc network scenario is to build up connections between nearest neighbors. If the mobility is very high, like on a crowded sidewalk, this leads to short communication links, that survive for only short time periods. Although it is possible to build up these connections and transmit some data, it is nearly impossible to maintain packet routes or maintain directories for efficient location of users. Therefore, we need communication links to sustain for some time span Δ to enable the routing layer to keep up with the dynamical changes. We can guarantee that a communication link between two moving stations sustains for this period if we adjust the transmission range to a value, which covers all possible distances the communication partners can reach in time Δ. Since, we know the maximum speed, this implies that the transmission power must be chosen such that the transmission range is at least $2\Delta v_{\max}$ larger than the distance at the beginning of the time interval. The task is now to appropriately build up the basic connection links such that the routing algorithm can choose routes with low energy or low congestion, while the number of edges and interfering edges is small.

A motivating example for the **acceleration bounded** model is given by **vehicles** of high speed, like cars, trains, or aircrafts. E.g., consider trains where each wagon carries a mobile radio station. Now consider a scenario, where two such trains pass each other in opposite directions, as shown in Figure 1. If we take a

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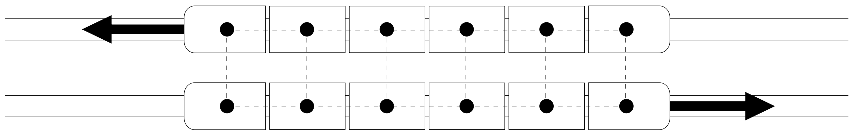

Figure 1: Snapshot of two trains passing each other in opposite directions.

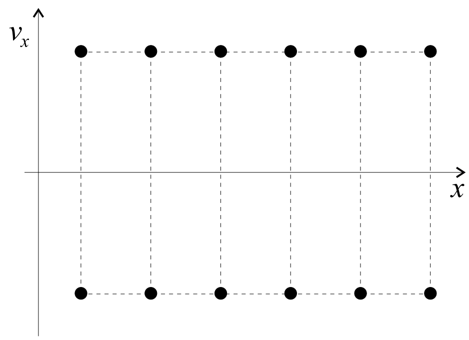

Figure 2: Horizontal speed and directions in the train example. Each point represents a wagon.

snapshot in this moment and build a static ad hoc network using the temporary positions, then this static approach may lead to a ladder-like network as shown in the figure. But the connection links forming the rungs of the ladder can be upheld only for a short time period since the trains move with high speed. After a short period all rung links need to be replaced by new ones. Therefore this static network design is not a good choice.

To generalize from linear movements to some worst-case settings we allow all nodes to accelerate by some maximum amount of $a_{\max}$, i.e. let $s_i''(t) = ds_i'(t)/dt$ (for $s_i'(t) = ds_i(t)/dt$ where $s_i(t)$ denotes the coordinates of a mobile station at time t) be the vector describing the acceleration of mobile station s_i, then we claim that $|s_i''(t)|_2 \leq a_{\max}$. Now, if we try to adjust the transmission range r of a connection such that the moving communication partners s_i and s_j of known relative speed $v = s_i'(t) - s_j'(t)$ and distance vector $d = s_i(t) - s_j(t)$ sustain for a time span Δ, we need a transmission distance that covers at least the distance d at the beginning and the distance at the end stretched by a possible acceleration, i.e. $r = \max\{|d|, |d + \Delta v|_2 + \Delta^2 a_{\max}\}$. In the train example this implies that the connection links between the passing trains are more expensive than one expects looking only at a snapshot. If we add to the two position coordinates the vertical and horizontal speed coordinates, we map the dynamical aspect of the scenario into four dimensions, as shown in Figure 2 (vertical speed and location coordinates are left out). In this setting the speed difference separates the trains. Hence, rung links between the trains add radio interferences with other edges. It is straight-forward that a small number of rung connections between the trains improve the mobile network, while the edges inside should follow a static ad hoc network policy.

Our goal is to build up a mobile ad hoc network that is stable and prefers short links. In a high-speed scenario one cannot provide both features at the same time. We will try to present a reasonable compromise. We allow any movement of the mobile stations within these restrictions and will compare the performance of our network solutions with the best offline solution for this dynamical scenario.

2 Previous Research

Many mobility models have been proposed as a basis for simulation of cellular and ad hoc networks. Most of them use a random process to vary speed or direction of the moving objects, like the *random walk* model an its variants. The most common model for simulations of cellular networks is a random walk model which describes mobility as a stop-and-go motion between cells. According to the *fluid flow* model every object moves with a randomly chosen speed and direction for a predefined time interval. In contrast to the random walk the motion is more predictable. The *Gauss-Markov* model [LH99] is a bit of both the random walk and the fluid flow model: Speed and direction are changed with an adjustable amount of randomness, ranging from completely random to predictable, linear motion.

In the *random waypoint* model [JM96] the objects move between randomly chosen positions where they pause for a certain time interval. Their speed is uniformly distributed between zero and a maximum. The speed chosen for the next motion period does not depend on the speed of the previous period. Thus sharp turns and sudden stops are possible, i.e. the acceleration is not bounded.

Besides these models, in which the movement of each object is independent from others, there are mobility models that regard mobility of a group of objects, e.g. the *reference point group mobility* model [HGPC99] that defines for groups of objects a logical center that determines direction, speed, and acceleration of each object. Other examples are the *column* model, the *pursue* model, and the *nomadic community* model. These models are less suitable to model worst case mobility as they provide some kind of smoothed or uniform mobility pattern. A survey of the mobility models mentioned above can be found in [CBD02, Bet01].

Chatzigiannakis et al. [CNS01] abstract from the geometric properties of movement and divide the motion space into cubes that approximate a sphere that is given by a predefined transmission range. These cubes are represented by the nodes of a motion graph, adjacent cubes are connected by an edge. Then mobility is modeled by a random walk on this motion graph.

Another way to deal with mobility in ad hoc networks is to consider what happens to the underlying topology when the nodes are moving. This leads to the *adversarial network model* [ABBS01] in which all communication links are under control of an adversary. A worst case for mobility corresponds with the topological changes the adversary may perform within some predefined restrictions.

In the context of computational geometry Basch et al. introduced the concept of *kinetic data structures* (KDS) [BGH99] that describes a framework for analyzing algorithms on mobile objects. In their model the mobility of objects is described by pseudo-algebraic functions of time and fully or partially predictable. The analysis of a KDS is done by counting the combinatorial changes of the geometric structure that is maintained by the KDS. Therefore the worst case mobility depends on the specific application for which the KDS is designed. Another approach that captures unpredictable mobility is the concept of *soft kinetic data structures* (SKDS) [CS01]. These data structures maintain an approximate geometric structure that is updated by property testing and reorganization. SKDS are evaluated with respect to the dynamics of the system, which is measured by the number of errors the data structure contains due to the mobility of objects. Worst case mobility is rather described as number of changes that violate the internal structure than as a random process. The mobility is regarded with respect to the specific purpose of the SKDS. It is not characterized in terms of velocity and direction.

The idea of kinetic data structures is also used in [GGH$^+$01a] to maintain a clustering of moving objects. This approach is used in [GGH$^+$01b] to determine the head of each cluster in a mobile network. In each cluster the nodes are directly connected to the head. The heads and some intermediate gateway nodes are connected by a Delaunay graph with restricted edge lengths that forms a backbone network. In this network routing can be performed by a geometric forwarding scheme. To react on mobility the clustering is updated by an event-based kinetic data structure.

3 The Model

In our model we consider a fixed set S of n mobile stations $s_1, \dots, s_n$ in the Euclidean plane. We denote by $s_i(t)$ the coordinates of a mobile station at time t and by $s_i'(t)$ the speed vector of s_i at time step t. Furthermore $s_i''(t)$ denotes the acceleration of s_i at time t, i.e. the change of the speed.

All mobile stations remain active all the time. We allow adjustable transmission power for each connection, which is high enough such that all mobile stations never leave the maximum transmission range of a mobile station. The mobile stations use omni-directional radio antennae, i.e. all mobile stations inside a disk with the sender as center and the transmission distance as radius can receive the message or will be disturbed while receiving data on a different connection. We assume bidirectional communication on a single frequency with time-multiplexing, i.e. using different time slots. Data need to be acknowledged and for simplicity we assume that the impact of acknowledgments is similar to the impact of sent data.

3.1 The Mobile Ad Hoc Network

We try to keep all connections alive for at least a fixed time interval of length Δ. This parameter is an over-all network parameter which induces some stability into the network. It should be chosen sufficiently large to set up the communication between neighbors, to update routing tables, and deliver some amount of data. For a practical realization it may not be necessary to adopt a synchronous round model as we will do now.

We assume that all nodes work synchronized in subsequent time intervals of length Δ. Then, during each time interval of length Δ the mobile ad hoc network performs the following operations.

1. Every mobile station determines the positions and speed vectors of possible (and reasonable) communication partners.
2. Every mobile station establishes communication links to some selected neighbors.
3. According to a routing algorithm basic routing information is computed, e.g. by routing tables, packet flooding, or diffusion algorithms.
4. The mobile network communicates data packets of the applications, i.e. telephone, e-mail, etc.

Note that this approach embodies the concept of a network protocol stack. The first phase refers to the physical layer, where physical data like transmission power and the change of the incoming signal can be used to estimate relative distances and relative speed. The second phase describes the task of the Medium Access Layer (MAC). Note that the specific routing requests are not known in this layer. Its task is to build up a general-purpose network which allows efficient routing, while the network graph is pruned such that the number of interfering edges is small.

In the third phase the routing algorithm can rely on a stable communication network for some time span Δ. Then, the routing in the mobile network is reduced to routing in a (temporary) static network and standard techniques are applicable. The packet routes are chosen to minimize latency, traffic-induced congestions and, typically for mobile devices, to reduce the transmission energy. In [MSVG02] it is shown that even in the static case it is not possible to optimize more than one of these parameters at the same time. However, in the static case it is possible to build up a general-purpose-network which enables the routing algorithm to choose its optimization policy afterwards. The fourth phase of our model describes the activity induced by the upper-most layer of the network protocol stack, the application layer.

In this paper we concentrate on the distributed computation of the interconnection network by the MAC layer and the problem of committing necessary location information in the physical layer.

3.2 Pedestrian Mobility

The **pedestrian mobility** model is a worst case approach relying on all mobile stations obeying a speed limit of $v_{\max}$. In this **velocity bounded** model the starting position $s_i := s_i(0)$ is known and for the speed vector $s_i'(t) = ds_i(t)/dt$ it holds $|s_i'(t)|_2 \leq v_{\max}$. This implies for the relevant time interval Δ that all mobile stations remain in a disk with radius $\Delta v_{\max}$ around the starting position s_i, i.e.

$$\text{for } t \in [0, \Delta] \ : \ |s_i(t) - s_i|_2 \ \leq \ \Delta \cdot v_{\max} \ .$$

3.3 Vehicular Mobility

The **vehicular mobility** model describes the movement of n stations with **bounded acceleration** $a_{\max}$. It refers to transportation vehicles which can operate at high speeds, where the limitation by the change of speed has a higher impact on the movement than the maximum possible speed, e.g. cars, trains, aircrafts. Let $s_i''(t) = ds_i'(t)/dt$ denote the acceleration vector of a mobile station s_i, then we claim that for all mobile stations $|s_i''(t)|_2 \leq a_{\max}$. Now, the starting speed vector $s_i' := s_i'(0)$ at the beginning of the time interval Δ can be arbitrarily large. Yet, we assume that at the beginning of the time interval $[0, \Delta]$ we know all locations $s_1, \ldots, s_n$ and all speed vectors $s_1', \ldots, s_n'$. Then, we can estimate the position of station i at time point $t \in [0, t]$ by

$$|s_i(t) - ts_i' - s_i|_2 \ \leq \ \frac{1}{2} a_{\max} t^2 \ \leq \ \frac{1}{2} a_{\max} \Delta^2 \ .$$

As a technical condition we require a polynomial bound on the maximum distances and relative speed differences for both models, i.e. for some constant k we claim $|s_i - s_j| \leq \mathcal{O}((\Delta v_{\max})^k)$ in the pedestrian model and $|s_i - s_j| + |s'_i - s'_j| \leq \mathcal{O}((\Delta a_{\max})^k)$ in the vehicular model.

4 Mobility and Network Parameters

In our worst-case approach scenarios may appear where even optimal networks have bad performance. To identify such scenarios we introduce a network independent measure, called **crowdedness**. We will see that it states a lower bound on the number of radio interferences and the maximum degree of reasonable connection networks.

For the **velocity bounded** model we define $\text{Crowd}_{\mathsf{v}}(u)$ of a node u by the set of all other nodes in distance $2v_{\max}\Delta$. Its cardinality defines $\text{crowd}_{\mathsf{v}}(u)$, the crowdedness of u. The overall-crowdedness of a set of stations S is given by the maximum crowdedness, i.e. $\text{crowd}_{\mathsf{v}}(S) := \max_{w \in S} \text{crowd}_{\mathsf{v}}(w)$.

In the **acceleration bounded** model we define the crowd of a node u by

$$\text{Crowd}_{\mathsf{a}}(u) := \left\{ w \in S \setminus \{u\} \; : \; |u - w|_2 \leq \frac{1}{2} a_{\max} \Delta^2 \text{ and } |u' - w'|_2 \leq \frac{1}{2} a_{\max} \Delta \right\} ,$$

where u, w denote the starting positions, and u', w' the starting vector of mobile stations for the time interval $[0, \Delta]$. The crowdedness $\text{crowd}_{\mathsf{a}}(u)$ is defined by its cardinality. It can be interpreted as the number of nodes that can approach u with maximum acceleration $a_{\max}$ in time Δ such that $s_i(\Delta) = s_j(\Delta)$ and $s'_i(\Delta) = s'_j(\Delta)$. The overall crowdedness is given by $\text{crowd}_{\mathsf{a}}(S) := \max_{u \in S} \text{crowd}_{\mathsf{a}}(u)$.

Transmission Range One crucial property of our mobile network approach is to build up persistent links for the time interval $[0, \Delta]$. The only method to ensure this property for a communication link is to increase the transmission distance such that all possible movements can be covered. In the velocity bounded **pedestrian model** we therefore redefine the transmission distance of two stations $u, w \in S$ by

$$|u, w|_{\mathsf{v}} := 2v_{\max}\Delta + |u - w|_2 \; .$$

In the **vehicular model** the following term describes the necessary transmission range.

$$|(u, u'), (w, w')|_{\mathsf{a}} := \max\{|u - w|_2, |u - w + (u' - w')\Delta|_2 + a_{\max}\Delta^2\} \; .$$

Note that both definitions are symmetric and fulfill the triangle inequality, i.e. $|a, b| + |b, c| \geq |a, c|$ (For a shorter notation we denote for the quadruple (u_x, u_y, u'_x, u'_y) simply u). Proofs for the correctness of these statements can be found in the Appendix.

The union of all (bi-directional) communication links E describes the mobile ad hoc network. The **diameter** $\text{diam}(G)$ of this undirected graph G is described by the maximum hop-distance between a pair of nodes. The **degree** $\deg(v)$ of a node v is the number of established communication links at v.

Interferences Modern communication networks use many frequencies and sophisticated spread spectrum techniques, that allow many senders to share the same medium. However, in most systems the bandwidth of the medium can be outnumbered by the communication load of the participants. For a theoretical approach we assume that such spread spectrum systems behave like a one-frequency network with a probabilistic time schedule.

In our one-frequency model with adjustable transmission distances edges interfere if a mobile station is inside the transmission area of an edge and messages are sent simultaneously [MSVG02]. Because of our little knowledge about the movement of the mobile stations, we cannot exactly predict interferences. For an accurate analysis one has to take into account the sending time of a message, the movements of senders and receivers, their transmission radii, the impact of control data induced by distance measurements and acknowledgments.

For a theoretical approach we need a simple definition that allows us to classify mobile networks. Radio interferences result from a combination of bad timing, bad locations and large transmission radii. We want to concentrate on the geographical cause of interferences and (pessimistically) count all interferences of communication links that **could** interfere at some time. In the static wireless network scenario this can be described by deciding, whether a node resides in the transmission area of a communication link. In our worst-case mobility scenario the situation is more difficult. We do not know whether the mobile station could move into the transmission area of a link. But we cannot assume that if disjoint communication areas are nearby that the mobile station will be interfered by all of them at the same time.

Therefore we (optimistically) define interferences as if the distance between two interfering stations is not stretched by the additional constant that is given by the velocity bound in the pedestrian model and the acceleration bound in the vehicular model. Thus, in our pedestrian model we count interferences as if the radio stations are not moving at all. For the vehicular model we count only interferences as if the interfering mobile stations are not accelerating (yet using a oversized transmission radius to compensate for worst-case movements). Furthermore, we count only interfering links if they interfere for the complete time span Δ.

In this interference model two edges do not interfere even if they pass each other at a close distance with high relative speed. One may argue that in this case the interaction between the links is so short that it can be neglected (Of course we are aware of worst-case scenarios of passing mobile clusters giving a counter-example). As a physical argument, the large relative speed difference may be large enough to cause a frequency shift, known as Doppler-effect, which prevents radio interferences.

This means for the **velocity bounded** model **an edge** $g = \{p_1, p_2\}$ **interferes with edge** $e = \{q_1, q_2\}$, i.e. $g \in \text{Int}_{\mathsf{v}}(e)$ if $\exists p_i \in e, \exists q_j \in g \ : \ |p_i - q_j|_2 \leq |g|_{\mathsf{v}}$.

In the **acceleration bounded** model we model interferences only between edges which interfere for the complete interval $[0, \Delta]$, if the velocity vectors of their nodes remain the same. Formally we define that edge $g = \{p_1, p_2\}$ interferes with edge $e = \{q_1, q_2\}$, i.e. $g \in \text{Int}_{\mathsf{a}}(e)$ if

$$\exists p_i \in e, \exists q_j \in g \ : \ |p_i - q_j|_2 \leq |g|_{\mathsf{a}} \quad \text{and} \quad |\tilde{p}_i - \tilde{q}_j|_2 \leq |g|_{\mathsf{a}} \ ,$$

where $\tilde{u} := u + \Delta u'$ denotes the position of u at time point Δ if the speed vector of u remained unchanged during $[0, \Delta]$. The interference number $\text{Int}_\alpha(G)$ of mobile network G is given by the maximum interfered set of edges ($\alpha \in \{\mathsf{a}, \mathsf{v}\}$):

$$\text{Int}_\alpha(G) := \max_{e \in E(G)} \{\text{Int}_\alpha(e)\} \ .$$

Now, the crowdedness of the underlying set of mobile station states a lower bound on the number of interferences every connected mobile network produces.

Theorem 1 *In both mobility models ($\alpha \in \{\mathsf{a}, \mathsf{v}\}$) we observe for all connected graphs $G = (S, E)$:*

$$\text{Int}_\alpha(G) \ \geq \ \text{crowd}_\alpha(S) - 1 \ .$$

Mobile Spanner A graph G is called a mobile spanner according to pedestrian or vehicular mobility ($\alpha \in \{\mathsf{a}, \mathsf{v}\}$) if for all nodes $u, w \in S$ there is a path $(u = p_0, p_1, \ldots, p_k = w)$ in G such that

$$\sum_{i=1}^{k} |p_{i-1}, p_i|_\alpha \ \leq \ c \cdot |u, w|_\alpha \ ,$$

for some constant c. For the optimization of the energy consumption we use the model that transmission power for sending to a distance d increases as a function d^β, where $\beta \geq 2$. (For the free space propagation model $\beta = 2$, for the so-called two ray model, which also considers multipath fading, $\beta = 4$.) Therefore, we define the notion of mobile power spanner analogously to the mobile spanner by replacing the last inequality with

$$\sum_{i=1}^{k} \left(|p_{i-1}, p_i|_\alpha\right)^\beta \ \leq \ c \cdot \left(|u, w|_\alpha\right)^\beta \ ,$$

Lemma 1 *For both mobility measures every mobile spanner is also a mobile power spanner.*

Proof: Consider a mobile spanner G and a path $(u = p_0, p_1, \ldots, p_k = w)$ in G.

$$\sum_{i=1}^{k} (|p_{i-1}, p_i|_\alpha)^\beta \leq \left(\sum_{i=1}^{k} |p_{i-1}, p_i|_\alpha\right)^\beta \leq (c \cdot |u, w|_\alpha)^\beta = c' \cdot (|u, w|_\alpha)^\beta$$

■

Congestion Following the approach in [MSVG02] we observe on each communication link e some packet **load** $\ell(e)$, which will be delivered in time interval $[0, \Delta]$. This load is caused by packets following routes (also called paths) which include e. The union of all these paths is called a path system $\mathcal{P}$.

As a worst case estimation on the number of packets that cause a congestion at link e we have to count all packets $\ell(e)$ as well as all packets being transported on interfering edges, which leads to the following definition of congestion $C_{\alpha,\mathcal{P}}(e)$ of an edge e with respect to a path system $\mathcal{P}$ for $\alpha \in \{\mathsf{a}, \mathsf{v}\}$:

$$C_{\alpha,\mathcal{P}}(e) := \ell(e) + \sum_{g \in \text{Int}_\alpha(e)} \ell(g) .$$

We describe the mobile network congestion by $C_{\alpha,\mathcal{P}}(G) := \max_{e \in E(G)} C_{\alpha,\mathcal{P}}(e)$.

If we know the optimal path system $\mathcal{P}$ in advance, the definition of the underlying optimal network is given by all edges used in the path system. However, because of the structure of the protocol stack we have to determine the network before knowing a path system or even routing requests. We solve this problem by showing that a mobile spanner hosts a path system that polylogarithmically approximates the optimal congestion.

Theorem 2 *Given a mobile spanner G then for every optimal path system $\mathcal{P}$ on a complete network C there exists a path system $\mathcal{P}'$ on G such that for $\alpha \in \{\mathsf{v}, \mathsf{a}\}$*

$$C_{\alpha,\mathcal{P}'}(G) = \mathcal{O}(C_{\alpha,\mathcal{P}}(C) \cdot \text{Int}_\alpha(G) \cdot \log n) .$$

The proof uses techniques presented in [MSVG02] in Theorem 5 and can be found in the Appendix.

5 Constructing Mobile Networks

Now we present techniques to construct mobile spanners with a small number of interferences. We use a **grid-cluster** technique, which adopts ideas of [AS98] for static ad hoc networks. For the pedestrian model we consider a grid of cell size $\Delta v_{\max}$. For every grid cell, where at least one mobile station resides at the beginning of the time interval $[0, \Delta]$, we elect one of them as a cluster head u_c. All other mobile stations in this cell have a connection link to the cluster head forming a star for each cell. The set of cluster heads S_c will be connected by a (static) spanner.

In the vehicular model, we consider a 4-dimensional grid $G_{i,j,k,\ell}$, where each cell forms a rectangle. A mobile station s_i with coordinates $(p_1, \ldots, p_4) = ((s_i)_x, (s_i)_y, (s'_i)_x, (s'_i)_y)$ is in the grid cell $q_1, \ldots, q_4 \in \mathbb{Z}^4$, i.e. $s_i \in G_q$, if $q_i = \lfloor \mathrm{g}^0(p)_i \rfloor$, where

$$\mathrm{g}^k(x, y, v_x, v_y) := \left(\frac{x 2^{-k}}{6 a_{\max} \Delta^2}, \frac{y 2^{-k}}{6 a_{\max} \Delta^2}, \frac{v_x 2^{-k}}{2 a_{\max} \Delta}, \frac{v_y 2^{-k}}{2 a_{\max} \Delta}\right) .$$

Like in the pedestrian model we elect a cluster head for each cell and a star-like communication network in each cell. All cluster heads will be connected by a (static) four-dimensional spanner.

Lemma 2 *The grid-cluster-technique constructs mobile spanners for both mobility models.*

Note that the spanning property does not imply a bound on the number of interferences. For this, we can use the Hierarchical Layer Graph construction presented in [MSVG02, LSVG02] as a spanner. Here, we use a simplified approach, called **Hierarchical Grid** (because of the knowledge of absolute coordinates).

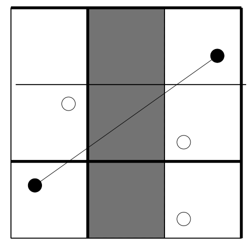

Figure 3: Separating rectangle for the hierarchical grid.

We start with the grid $G^0 = G$ introduced above and the set of all stations $S_0 := S$, and promote for each cell one mobile station for being cluster head. The set of these cluster heads form the set S_1. In this level only cluster heads may communicate with nodes in their cell. Besides these links, cluster heads have communication links with all cluster heads in neighbored cells sharing at least a corner of the grid.

We iterate this extended grid-cluster-technique until only one point is left. Formally, in the i-th level of the network structure, we start with a set of stations S_i and consider the grid G^i, with grid coordinates $\mathrm{g}(s_j)$. A cell $q \in \mathbb{Z}^4$ contains all points p with $\lfloor \mathrm{g}^i(p)_\ell \rfloor = q_\ell$. We assign for each cell a cluster head and add it to the set S_{i+1}. We connect each of the cluster heads to all nodes of rank i in its cell. Then, we connect all cluster heads to all cluster heads of neighbored cells. For the pedestrian model we use an analogous construction based on the two-dimensional grid cell size $2v_{\max}\Delta$.

Theorem 3 *For both mobility models the Hierarchical Grid Graph constitutes a mobile spanner with at most $\mathcal{O}(\mathrm{crowd}_\alpha(V) + \log n)$ interferences.*

Proof: We start with the proof that the Hierarchical Grid Graph is a mobile spanner. If two nodes lie in the same grid cell of G^0, then they have hop-distance of at most 2, since both of them are connected with the cluster head. Since $|u,v|_{\mathsf{a}} \geq a_{\max}\Delta^2$ and $|u,v|_{\mathsf{v}} \geq 2v_{\max}\Delta$ and the cell size is linear in this distance, this case is settled.

We now prove that the mobile spanning property holds with respective to the grid distance measure $|\mathrm{g}^0(u), \mathrm{g}^0(w)|_2$. We recursively define a path from u to w using separating (two or four-dimensional) rectangles. A separating rectangle R of level i consists only of empty grid cells in G^i, such that the entry points of line (u,w) in R have distance (in L_2-norm) of at least one cell size according to the grid G^i, i.e. 2^i with respect to G^0 (Figure 3).

Let i be the highest level where such $k \geq 1$ separating rectangles exist for given mobile stations u and w. Let $L(d)$ denote the length of a path connecting u and w in the Hierarchical Grid Graph, for $d = |\mathrm{g}^0(u), \mathrm{g}^0(w)|_2$. Then, for each of the k separating rectangles the maximum width is given by 2^{i+1}. Furthermore, there exist cluster heads of level $i+1$, where p_j is on the side nearer to u and q_j on the side nearer to w for $j \in \{1, \ldots, k\}$ starting from u (since otherwise, i would not have been the highest level with separating rectangles). It is straight-forward that there are short paths between q_j and p_{j+1} that use only cluster heads of G_i. The overall sum of these paths is linear in the distance d, likewise the distance of all edge distances $\sum_{j=1}^{k} |\mathrm{g}^0(p_j), \mathrm{g}^0(q_j)|_2$.

For a recursion we have to look at the path between u and p_1, and the analogous case of the path between q_k and w. Clearly, in the grid cell of u in G^i there exists a cluster head v and v is connected via a path using only cluster heads of level i to p_1. Note that u and v are in the same grid cell of level G^i and therefore u and v are in neighbored cells of the level $i-1$. Hence, the next lower separating rectangle between u and v can be at most in level $i-2$. This leads to the recursion $L(d) \leq c \cdot d + 2L\left(\frac{d}{4}\right)$, for a constant c. Thus, $L(d) = O(d)$. It remains to show that

$$|\mathrm{g}^0(u), \mathrm{g}^0(w)|_2 = \Theta(|u,w|_{\mathsf{a}}) \ , \quad \text{for } |\mathrm{g}^0(u), \mathrm{g}^0(w)|_2 \geq 1 \qquad (1)$$

(if we consider Δ, $a_{\max}$, resp. $v_{\max}$ as constants). This can be done using elementary algebra.

For the number of interferences, it is essential that every edge e connecting neighbored cells in G^i only interferes with an edge e', if for a node u of e and w of e' it holds $|\mathrm{g}^i(u), \mathrm{g}^i(w)|_2 = O(1)$. This follows directly from (1). Hence, most of the interferences occur at the lowest level of the grid G^0, where a mobile station can suffer under the radio interferences of at most $O(\mathrm{crowd}_\alpha(S))$ mobile stations in some constant number of near cells. In every higher grid level this amount reduces to a constant number of radio interferences, because only one cluster head resides in a cell and the number of connections with cluster heads of next lower or same level is constant. The assumption at the end of Section 3 implies that we have at most $O(\log n)$ grid levels, which completes the proof. ■

6 Position Information Management

In the previous sections, we have assumed that all positioning information is available to all nodes. But distributing this information is a non-trivial task. This process has to be done in the physical layer, where routing is not available. But the physical layer may use a positioning system (e.g. GPS) which enables every mobile station to learn its position and absolute speed. Then, it can use short radio broadcast messages, called beacons, to inform all neighbors in the transmission range.

Another solution is to measure the distances between mobile stations by comparing incoming and outgoing transmission power. On first sight this looks similar to the position beacon model. However, using only distance information it is impossible to compute the relative speed of the communication partner.

We dedicate this section to present solutions for dynamic position information management under the vehicular mobility model (which can be easily applied to the pedestrian mobility model as well).

6.1 Coordinating Location Beacons

The physical layer is responsible for the distribution of relative positions. If the mobile stations can determine their absolute coordinates (and thus can compute absolute speed vectors), this information must be broadcasted to construct the basic network of the next round (of time span Δ).

A straightforward solution is sending special beacon signals carrying location and identification information. One can assign special time slots for these beacon signals, which are not propagated by other nodes. However, if all nodes send these beacons at maximum transmission range, then n beacon signals would interfere at each node. It turns out that a data structure as the Hierarchical Grid helps to reduce the necessary transmission range and the size for the beacon time slot. For this, we use the following observation.

Lemma 3 *Let s_i, s_j be mobile stations under vehicular mobility and let $\tilde{s}_i := s_i(\Delta)$, $\tilde{s}_j := s_j(\Delta)$. Then, for $k > 0$*

$$\frac{2}{3}|\mathrm{g}^k(s_i), \mathrm{g}^k(s_j)|_\infty - 2^{-k} \le\ |\mathrm{g}^k(\tilde{s}_i), \mathrm{g}^k(\tilde{s}_j)|_\infty\ \le\ \frac{4}{3}|\mathrm{g}^k(s_i), \mathrm{g}^k(s_j)|_\infty + \frac{1}{6}2^{-k}\ .$$

Proof Sketch: The relative change of the position coordinates $x(s_i) - x(\tilde{s}_i)$ is bounded by at most $\pm(\Delta \cdot |(x(s_i') - x(s_j')| + a_{\max}\Delta^2)$, while the relative change of the velocity coordinate $x(s_i') - x(\tilde{s}_i')$ is bounded by $\pm 2\Delta a_{\max}$ and analogously for the y-coordinates. The rest of the proof follows straightforward by a case study. ■

This observation helps to bound the dynamic changes of the Hierarchical Grid Graph. If a node u has rank $i > 1$ in round t, then the number of nodes of equal rank in the same cell as u is bounded by 9 in the pedestrian model and by 81 in the vehicular model.

Using this observation, we currently are working on an efficient data structure based on the Hierarchical Grid where the movement of a node in the hierarchical data structure is bounded. Our goal is to find a selection strategy for assigning cluster heads such that a cluster head of rank i is assigned at most rank $i + 1$ in the next round. This would have implications for the necessary transmission range of the location beacons. To get the information for coordinating cluster heads in the next round each node of rank i can use a transmission range which is only a constant factor larger than those of adjacent edges. As a consequence, the number of time slots occupied by the beacon signals can be reduced significantly.

6.2 Distances as Location Information

There is a number of reasons why absolute coordinates in mobile stations are not available, e.g. size and cost of GPS subsystems, reachability, accurateness. However, we will show that little relative distance information is sufficient to maintain a good mobile network structure. We assume, that a mobile station can measure the distance to another station by measuring the receiving transmission power. Such measurements can be performed in the physical layer of the protocol stack and we reduce the number of measurements, while still maintaining a mobile network similar to a grid-cluster network. Define $\delta_{i,j}(t) = |s_i(t) - s_j(t)|_2$.

Lemma 4 *Given distances $\delta_{i,j}(-\Delta)$ and $\delta_{i,j}(0)$ it is possible to approximate the transmission distance $|s_i, s_j|_{\mathsf{a}}$ by a factor of 3.*

Proof: We use $\tilde{\delta} := \max\{\delta_{i,j}(0), |2\delta_{i,j}(0) - \delta_{i,j}(-\Delta)| + \frac{1}{2}a_{\max}\Delta^2\}$ as an approximation for $|s_i, s_j|_{\mathsf{a}}$ and show that $\tilde{\delta} \leq |s_i, s_j|_{\mathsf{a}} \leq 3\tilde{\delta}$. Assume s_i measures $s_j(-\Delta)$ and $s_j(0)$. Then s_i can estimate the velocity $s'_j(0)$ by $|\delta_{i,j}(0) - \delta_{i,j}(-\Delta)|$. This value represents the average speed during $[-\Delta, 0]$. Let $d := \delta_{i,j}(0)$ the distance and $v := |s'_i(0) - s'_j(0)|_2$ the relative velocity at time 0. A worst case for unaccelerated movement occurs, if s_j passes s_i such that $\delta_{i,j}(-\Delta) = \delta_{i,j}(0) = d$ and $v = 2d/\Delta$. Then s_i cannot distinguish $\delta_{i,j}(-\Delta)$ from $\delta_{i,j}(0)$ and estimates a transmission distance of d. Since $\delta_{i,j}(\Delta) = 3d$ the distance is underestimated by a factor of at most 3. Assume s_j accelerates with $a_{\max}$ during $[0, \Delta]$, then the velocity $s'_j(\Delta) = 2a_{\max}$. The distance s_j has covered during $[-\Delta, 0]$ is $\frac{1}{2}a_{\max}\Delta^2$ and the distance during $[0, \Delta]$ is $\frac{3}{2}a_{\max}\Delta^2$. A worst case location for s_i is in the middle between $s_j(-\Delta)$ and $s_j(0)$. Then $\tilde{\delta} = \frac{1}{4}a_{\max}\Delta^2 + \frac{1}{2}a_{\max}\Delta^2 = \frac{3}{4}a_{\max}\Delta^2$, whereas $|s_i, s_j|_{\mathsf{a}} = |\frac{1}{4}a_{\max}\Delta^2 + a_{\max}\Delta^2| + a_{\max}\Delta^2 = \frac{9}{4}a_{\max}\Delta^2$ because the velocity $s'_j(0) = a_{\max}\Delta$. I.e. the transmission distance is underestimated by a factor of at most 3. Now, we show that $\tilde{\delta} \leq |s_i, s_j|_{\mathsf{a}}$. Because $\delta_{i,j}(0) = |s_i, s_j|_2$ we only have to show that $|2\delta_{i,j}(0) - \delta_{i,j}(-\Delta)| + \frac{1}{2}a_{\max}\Delta^2 \leq |d + v\Delta| + a_{\max}\Delta^2$. We know that for unaccelerated movement the distance $\tilde{\delta}$ is always underestimated. So we have to consider the error of the estimation for accelerated movement. If s_j accelerates, e.g. $s'_j(-\Delta) = 0$ and $s'_j(0) = a_{\max}\Delta$, then the error of the estimation is $\pm\frac{1}{2}a_{\max}\Delta$. Thus $|\delta_{i,j}(0) - \delta_{i,j}(-\Delta)| = |d + v\Delta \pm \frac{1}{2}a_{\max}\Delta|$ and $\tilde{\delta} \leq |s_i, s_j|_{\mathsf{a}}$. ■

Based on the physical restriction on the movements of mobile stations, we have found a dynamic data structure, called **Mobile Hierarchical Layer graph** (MHL-graph). Essentially, it uses the same ideas as the Hierarchical Grid graph, if one replaces the grid distance measure with the Euclidean L_2-norm. Again, one can show that this distance measure can be approximated by two distance measurements at different time points. The notion of cells will be replaced by a disk around nodes of certain rank. These MHL-graphs are mobile spanners and approximate the minimum number of mobile interferences by a constant factor.

Currently, we are working on a distributed algorithm that updates the Hierarchical Grid using only $\mathcal{O}(\text{crowd}_{\mathsf{a}}(V) + \log n)$ distance measurements per node at time 0 and $-\Delta$. This means that the Hierarchical Grid of the current round initiates distance measurements between dedicated nodes, that provide the necessary information to construct the Hierarchical Grid structure of the next round.

7 Conclusion and Open Problems

We have discussed two worst case models for mobility. In the first pedestrian motivated model we bound the speed by a speed limit of $v_{\max}$. In the other model designed for the special mobility induced by high-speed vehicles we assume that the acceleration of all nodes is bounded by a constant $a_{\max}$. To ensure some elementary stability in a wireless ad hoc network we adjust the transmission range of senders such that we can guarantee the persistence of all communication links for at least a time period of length Δ.

For the network construction we concentrate on the medium access layer, which builds up communication links without known routing tasks. We have presented a distributed algorithm to build up such an elementary mobile network, which allows small congestion, few interferences, low energy data routes, small degree and small diameter as summarized in the following corollary.

Corollary 1 *There exist distributed algorithms that construct mobile networks for the velocity bounded and the acceleration bounded mobility model with the following properties:*

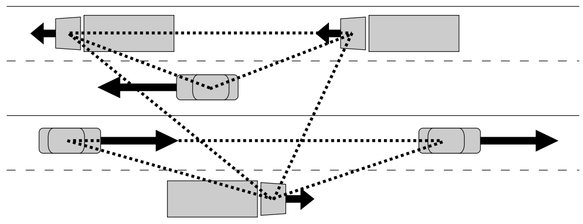

Figure 4: Vehicles on a highway with different speed vectors

1. *The mobile networks allows data routes on this mobile network inducing a congestion of at most* $O(\log^2 n)$ *times the congestion of the optimal routing.*

2. *The interference number of the mobile network approximates the optimal interference number by a factor of* $O(\log n)$.

3. *Energy-optimal routes can be approximated by a constant factor.*

4. *The degree is bounded by* $O(\text{crowd}_\alpha(V) + \log n)$ *and the diameter is at most* $O(\log n)$.

For the routing problem this does not imply that small congestion, low energy and short routes can be optimized by the same routing policy. Already for the static case of wireless networks one experiences trade-offs between any two of these measures [MSVG02]. However, the algorithms presented here, build up a general-purpose communication network, which performs well for all kinds of routing requests and has reasonable approximation ratios for any routing policy concerning congestion, energy or dilation.

As a side effect of our network construction for the vehicular mobility model we achieve a data structure, where clustering takes locality of positions and movement into account. E.g. consider a highway of two lanes in each direction, see Figure 4. All algorithms and data structures, neglecting the impact of relative speed, build up too many communication links between the opposing lanes. However, these links are very unstable and hence expensive. In our model communication links along each direction are much more frequent and, if there is a choice, then the communication link across the middle of the highway will be established between slower moving vehicles (e.g. trucks) instead faster ones (and this is even the case if the fast vehicles are nearer to each other).

There is a number of **open questions** left by this first approach to velocity and acceleration bounded mobility. First the modeling of interferences of moving edges, presented here, is only a rough estimation. It is not clear how a mobile network design may look for a more accurate model.

The movements of the communication partners also affects the routing algorithms. We have neglected the problem of long routes, that need more time than the update time Δ. Possibly, communication can be improved if for the communication paths those nodes are preferred that also move towards the receiver's direction.

The distributed measurement and computation of the relative locations and relative speed vectors states a problem. We have seen that a rough approximation can be deduced from the receiving transmission energy and its change in time. However, for the construction of a reasonable basic network, a direct implementation of previous approaches needs some interaction between location beacons and distance measurements. We are currently working on an efficient update strategy for mobile spanners such that the number of these interactions is minimized.

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8 Appendix

8.1 Proof of the triangle inequality for the distance measures

For the **velocity bounded** model the symmetry and the triangle inequality of the distance measure follows from the symmetry and triangle inequality of the Euclidean norm. For the **acceleration bounded** model the symmetry of the distance measure also follows from the symmetry of Euclidean norm. We prove the triangle inequality:

First, we define $|\cdot|_{\mathsf{a}}^*$ and proof the triangle inequality:

$$\begin{aligned} |u-w|_{\mathsf{a}}^* &:= |u-w+(u'-w')\Delta|_2 + a_{\max}\Delta^2 \\ &= |u-v+v-w+(u'-v'+v'-w')\Delta|_2 + a_{\max}\Delta^2 \\ &\leq |u-v+(u'-v')\Delta|_2 + |v-w+(v'-w')\Delta|_2 + a_{\max}\Delta^2 \\ &\leq |u-v+(u'-v')\Delta|_2 + a_{\max}\Delta^2 + |v-w+(v'-w')\Delta|_2 + a_{\max}\Delta^2 \\ &= |u-v|_{\mathsf{a}}^* + |v-w|_{\mathsf{a}}^* \end{aligned}$$

Then, we prove the triangle inequality for the the distance measure $|u,w|_{\mathsf{a}}$.

$$\begin{aligned} |u,w|_{\mathsf{a}} &= \max\{|u-w|_2, |u-w|_{\mathsf{a}}^*\} \\ &= \max\{|u-v+v-w|_2, |u-v+v-w|_{\mathsf{a}}^*\} \\ &\leq^{\dagger} \max\{|u-v|_2+|v-w|_f, |u-v|_{\mathsf{a}}^*+|v-w|_{\mathsf{a}}^*\} \\ &=^{\ddagger} \frac{1}{2}\Big||u-v|_2+|v-w|_2+|u-v|_{\mathsf{a}}^*+|v-w|_{\mathsf{a}}^*\Big|_2 \\ &\quad +\frac{1}{2}\Big||u-v|_2+|v-w|_2-|u-v|_{\mathsf{a}}^*-|v-w|_{\mathsf{a}}^*\Big|_2 \\ &\leq \frac{1}{2}\Big||u-v|_2+|u-v|_{\mathsf{a}}^*\Big|_2 + \frac{1}{2}\Big||v-w|_2+|v-w|_{\mathsf{a}}^*\Big|_2 \\ &\quad +\frac{1}{2}\Big||u-v|_2-|u-v|_{\mathsf{a}}^*\Big|_2 + \frac{1}{2}\Big||v-w|_2-|v-w|_{\mathsf{a}}^*\Big|_2 \\ &= \max\{|u-v|_2, |u-v|_{\mathsf{a}}^*\} + \max\{|v-w|_2, |v-w|_{\mathsf{a}}^*\} \\ &= |u,v|_{\mathsf{a}} + |v,w|_{\mathsf{a}} \end{aligned}$$

† follows from the triangle inequality for $|\cdot|_2$ and $|\cdot|_{\mathsf{a}}^*$.

‡ $\max\{a,b\} = \frac{1}{2}|a+b| + \frac{1}{2}|a-b| \;\forall a,b\in\mathbb{N}_0$ ■

8.2 Proof of Theorem 1

Let $u \in S$ be a node for which $\mathrm{crowd}_\alpha(S)$ is maximal in G and let e be an arbitrary edge incident to u. Such an edge exists, because G is connected. We define the set $E(\mathrm{Crowd}_\alpha(u))$ as the set of of edges incident to a node from $\mathrm{Crowd}_\alpha(u)$, i.e. $E(\mathrm{Crowd}_\alpha(u)) := \{g = \{w,z\} : w \in \mathrm{Crowd}_\alpha(u)\}$. Consider the Graph $\hat{G}$ which is obtained from G by substituting each node of $S \setminus \mathrm{Crowd}_\alpha(u)$ by a new node $\hat{u}$. (The position of $\hat{u}$ is irrelevant.) The edges which are incident to a node from $S \setminus \mathrm{Crowd}_\alpha(u)$ in G become incident to $\hat{u}$ in $\hat{G}$ (multiple edges are deleted). Note, that $u \in S \setminus \mathrm{Crowd}_\alpha(u)$, so $S \setminus \mathrm{Crowd}_\alpha(u)$ is not empty. Therefore the number of nodes in $\hat{G}$ is $\mathrm{crowd}_\alpha(u)+1$. Furthermore each edge in $\hat{G}$ is incident to a node of $\mathrm{Crowd}_\alpha(u)$ and so contained in $E(\mathrm{Crowd}_\alpha(u))$. Since G is connected, the graph $\hat{G}$ is also connected. Hence, the number of edges of $E(\mathrm{Crowd}_\alpha(u))$ in $\hat{G}$ is at least $\mathrm{crowd}_\alpha(u)$. Since each edge in $\hat{G}$ is incident to a node of $\mathrm{Crowd}_\alpha(u)$, it is contained in $E(\mathrm{Crowd}_\alpha(u))$. Hence $|E(\mathrm{Crowd}_\alpha(u))| \geq \mathrm{crowd}_\alpha(u)$. We show that each of the edges in $E(\mathrm{Crowd}_\alpha(u)) \setminus \{e\}$ is interfering with e. This will imply that $\mathrm{Int}_\alpha(G) \geq \mathrm{crowd}_\alpha(u) - 1 = \mathrm{crowd}_\alpha(S) - 1$.

In the velocity bounded model $\mathrm{Crowd}_{\mathsf{v}}(u)$ is defined as the set $\{w \in S : |u-w|_2 \leq 2v_{\max}\Delta$ and $w \neq u\}$. Then each edge $g = \{w,z\} \in G$ incident to a node $w \in \mathrm{Crowd}_{\mathsf{v}}(u)$ interferes with e, because $|u-w|_2 \leq |g|_{\mathsf{v}} + 2v_{\max}\Delta$ holds. Therefore, $E(\mathrm{Crowd}_{\mathsf{v}}(u)) \setminus \{e\} \subseteq \mathrm{Int}_{\mathsf{v}}(e)$.

In the acceleration bounded model $\mathrm{Crowd}_{\mathsf{a}}(u)$ is defined as the set $\{w \in S : |u-w|_2 \leq \frac{1}{2}a_{\max}\Delta^2$ and $|u'-w'|_2 \leq \frac{1}{2}a_{\max}\Delta$ and $w \neq u\}$. For each edge $g = \{w,z\} \in G$ incident to a node $w \in \mathrm{Crowd}_{\mathsf{a}}(u)$

interferes with e, because $|u-w|_2 \le |g|_{\mathsf{a}} = \max\{|u-w|_2, |u-w+(u'-w')\Delta|_2 + \Delta^2 a_{\max}\}$ holds. Hence $E(\mathrm{Crowd}_{\mathsf{a}}(u)) \setminus \{e\} \subseteq \mathrm{Int}_{\mathsf{a}}(e)$.

8.3 Proof of Lemma 2

For a given mobile station u let $h(u)$ denote the cluster head of the cell where u is located. We choose as a path for two given nodes u, w the shortest path between $h(u)$ and $h(w)$ using only cluster heads combined with the edges $(u, h(u))$ and $(h(w), w)$.

For the **velocity bounded mobility** model the transmission distance between a node and its cluster head is at most $(\sqrt{2}+1)\Delta v_{\max}$. The number of hops h between the cluster heads is linearly bounded by the Manhattan-distance between u and w according to the grid.

Hence, the additional impact of every hop of the path is linearly bounded by the distance. In the case that the two nodes are very near, i.e. $|u,w|_2 \le \Delta v_{\max}$, one uses that $|u,w|_{\mathsf{a}} \ge 2\Delta v_{\max}$. Because of the nearness of the cluster heads, there is only a constant number of hops and a linear long detour with respect to $2\Delta v_{\max} \le |u,w|_{\mathsf{a}}$.

For the **acceleration bounded mobility** model an analogous argumentation applies.

8.4 Proof of Theorem 2

Let $K_{\mathsf{v}} := 2\Delta v_{\max}$ and $K_{\mathsf{a}} := \Delta^2 a_{\max}$. Let $d(\mathsf{v}) = 2$, $d(\mathsf{a}) = 4$. In this proof we denote for the vehicular measure we represent a node u by

$$u = (u_1, \dots, u_4) = \left(\left(s_i - \frac{\Delta s_i'}{2} \right), \left(s_i + \frac{\Delta s_i'}{2} \right) \right) .$$

Then $|u,w|_{\mathsf{a}} = \max\{|u_{1,2}, w_{1,2}|_2, |u_{3,4}, w_{3,4}|_2 + K_{\mathsf{a}}\}$, where $u_{1,2} := (u_1, u_2)$. We define the metric $|\cdot|_{\mathsf{am}}$ as

$$|u,w|_{\mathsf{am}} := \max\{|u_{1,2}, w_{1,2}|_2, |u_{3,4}, w_{3,4}|_2\} .$$

The relationship between this metric and the transmission distance is the following.

$$|u,w|_{\mathsf{a}} - K_{\mathsf{a}} \le |u,w|_{\mathsf{am}} \le |u,w|_{\mathsf{a}} ,$$

while the relationship between L_2 and the transmission distance in the pedestrian model is

$$|u,w|_{\mathsf{v}} = |u,w|_2 + K_{\mathsf{v}} .$$

Define the interference region $D_\alpha(e)$ of an edge e as the set of points, which can be interfered by an edge e, i.e.

$$\begin{aligned} D_{\mathsf{v}}(e) &:= \{x \in \mathbb{R}^{d(\alpha)} \mid \exists p \in e \ : \ |x-p|_2 \le |e|_{\mathsf{v}}\} , \\ D_{\mathsf{a}}(e) &:= \{x \in \mathbb{R}^{d(\alpha)} \mid \exists p \in e \ : \ |x-p|_{\mathsf{am}} \le |e|_{\mathsf{a}} \} . \end{aligned}$$

For the vehicular distance measure we need the following Lemma.

Lemma 5 *There are $c_{\mathsf{a}} \le 72$ disjoint sub-spaces $A_1, \dots, A_{k_\alpha} \subset \mathbb{R}^4$ such that*

$$\forall i \in \{1, \dots, k_\alpha\}\ \forall u, p, w \in A_i \ : \ |u,w|_{\mathsf{am}} \le |u,p|_{\mathsf{am}} \implies |p,w|_{\mathsf{am}} \le |u,p|_{\mathsf{am}} .$$

Proof: Without loss of generality let $u = (0,0,0,0)$. Define for $k, j \in \{1, \dots, 36\}$:

$$\begin{aligned} A_{6j+k-6} &:= \Big\{ x \in \mathbb{R}^4 \mid \measuredangle(x_1,x_2) \in \Big[k\frac{\pi}{3}, (k+1)\frac{\pi}{3}\Big) \\ &\qquad \text{and } \measuredangle(x_3,x_4) \in \Big[j\frac{\pi}{3}, (j+1)\frac{\pi}{3}\Big) \text{ and } |x_1,x_2|_2 \ge |x_3,x_4|_2 \Big\} , \\ A_{36+6j+k-6} &:= \Big\{ x \in \mathbb{R}^4 \mid \measuredangle(x_1,x_2) \in \Big[k\frac{\pi}{3}, (k+1)\frac{\pi}{3}\Big) \\ &\qquad \text{and } \measuredangle(x_3,x_4) \in \Big[j\frac{\pi}{3}, (j+1)\frac{\pi}{3}\Big) \text{ and } |x_1,x_2|_2 < |x_3,x_4|_2 \Big\} . \end{aligned}$$

where $\measuredangle(a,b)$ denotes the angle between vector (a,b) and $(1,0)$. Let $p,w \in A_1$. Then, $|0,p|_{\mathsf{am}} = |p_1,p_2|_2$ and $|0,w|_{\mathsf{am}} = |w_1,w_2|_2$. Assume $|w_1,w_2|_2 \le |p_1,p_2|_2$. Then, by a straight-forward geometric argument $|(p_1,p_2)-(w_1,w_2)|_2 \le |p_1,p_2|_2$.

Since $|w_3,w_4| \le |w_1,w_2| \le |p_1,p_2|$ and $|p_3,p_4| \le |p_1,p_2|$ it follows that $|(p_3,p_4)-(w_3,w_4)|_2 \le |p_1,p_2|$. This implies $|p,w|_{\mathsf{am}} = \max\{|(p_1,p_2)-(w_1,w_2)|_2, |(p_3,p_4)-(w_3,w_4)|_2\} \le |p_1,p_2|$.

This argumentation also applies for all the other sub-spaces. ■

We extend the notion of congestion to nodes by counting all traffic which send out radio interferences to the location of the point:

$$C_{\alpha,\mathcal{P}}(u) := \sum_{e\in E(\mathcal{P}):u\in D_\alpha(e)} \ell(e) \ .$$

For an edge $e=(u,w)$ the following relationship is valid.

$$\max\{C_{\alpha,\mathcal{P}}(u), C_{\alpha,\mathcal{P}}(w)\} \ \le\ C_{\alpha,\mathcal{P}}(e) \ \le\ C_{\alpha,\mathcal{P}}(u) + C_{\alpha,\mathcal{P}}(w) \ .$$

The maximum congestion of any point in $\mathbb{R}^d$ is linearly bounded by the congestion of the network.

Lemma 6 *For all graphs $G=(V,E)$ with $V \subset \mathbb{R}^{d(\alpha)}$, all path systems $\mathcal{P}$ and all points $x \in \mathbb{R}^{d(\alpha)}$:*

$$C_\alpha(x) = \sum_{e\in E(G):x\in D_\alpha(e)} \ell(e) \ \le\ k_\alpha \cdot \max_{e\in E(G)} \sum_{e'\in \mathit{Int}(e)} \ell(e') = k_\alpha C_\alpha(\mathcal{P}) \ ,$$

for constants $k_\alpha > 1$.

Proof: For the point x we partition the space into c_α disjoint sub-spaces $A_1,\dots,A_{c_\alpha}$ such that for all $u,v \in A_i$ $|u,x|_\alpha \le |v,x|_\alpha$ implies $|u,v|_\alpha \le |v,x|_\alpha$ for $\alpha \in \{2,\mathsf{am}\}$. For the pedestrian mobility model, then the angle between $\overline{xu}$ and $\overline{xv}$ less or equal than $\pi/3$. Clearly, the optimal choice is $c_{\mathsf{v}} = 6$, which resembles six sectors centered at x. For the vehicular model, it follows by Lemma 5 that $c_{\mathsf{a}} \le 72$ suffices. Now choose for each sub-space A_i a vertex $u_i \in A_i$ that minimizes the distance $|x,u_i|_\alpha$ (if the sub-space is not empty). For every edge (v,w) with $x \in D_\alpha(\{v,w\})$ we now show that there exists a vertex u_i with $u_i \in D_\alpha(\{v,w\})$.

Assume that $x \in D_\alpha(\{v,w\})$ and let A_i be in the sub-space where v lies in. Since $|u_i,x|_\alpha \le |x,v|_\alpha$ we have $|u_i,v|_\alpha \le |x,v|_\alpha \le |v,w|_\alpha$. Therefore we have

$$\begin{aligned}
\sum_{e\in E(G):x\in D_\alpha(e)} \ell(e) &\le \sum_{i=1}^{c_\alpha} \sum_{e\in E(G):u_i\in D_\alpha(e)} \ell(e) \\
&\le c_\alpha \cdot \max_{u\in V(G)} \sum_{e\in E(G):u\in D_\alpha(e)} \ell(e) \\
&\le c_\alpha \cdot \max_{e\in E(G)} \sum_{e'\in \mathrm{Int}(e)} \ell(e') \ .
\end{aligned}$$

■

Lemma 7 *If all transmission radii of a mobile network are increased by a constant factor, then the congestion increases by at most a factor of $O(\log(V))$, if for all mobile stations hold $|s_i - s_j| + |s'_i - s'_j| \le \mathcal{O}((\Delta a_{\max})^k)$ in the vehicular model and $|s_i - s_j| \le \mathcal{O}((\Delta v_{\max})^k)$ in the pedestrian model.*

Proof: First we consider the path system $\mathcal{P}_r$ which consists of all sub-paths of $\mathcal{P}$ which consists only of edges with transmission distance in the range $[r,2r)$ for $r \ge k_\alpha$. Clearly, $C_{\alpha,\mathcal{P}_r}(u) \le C_{\alpha,\mathcal{P}}(u)$.

Let u be a node, which maximizes $C_{\alpha,\mathcal{P}_r}(u)$. Now we place c points $p_1,\dots,p_c$ in the surrounding of u such that for $\alpha \in \{2,\mathsf{am}\}$

$$\forall x \in \mathbb{R}^d \quad \exists i \in \{1,\dots,c\} \ :\ |p_i - x|_\alpha \le r \quad \text{and} \quad \forall i \in \{1,\dots,c\} \quad |p_i - x|_\alpha \le 2cr \ .$$

Such an arrangement of a constant number c of points can be achieved by a grid placement of p_i. Now, every edge that interferes with a point u with transmission radius $w \le 2cr$ has had an interference with a point p_i, when the transmission radius was in the interval $[r, 2r)$. This implies for the congestion $C'_{\alpha,\mathcal{P}_r}(u)$ of a point for a network where the transmission range is c times higher than necessary the following.

$$C'_{\alpha,\mathcal{P}_r}(u) \leq C_{\alpha,\mathcal{P}_r}(u) + \sum_{i=1}^{c} C_{\alpha,\mathcal{P}_r}(p_i) \ \leq \ (c+1)k_\alpha C_{\alpha,\mathcal{P}_r}(G) \ .$$

There are $O(\log n)$ different intervals $[r, 2r)$ that cover all available transmission distances. Each of these intervals induces congestion of at most $(c+1)k_\alpha C_{\alpha,\mathcal{P}_r}(G)$. Summing up these terms proves the claim. ■

Lemma 8 *Let C^* be the congestion of a given (congestion-optimal) path system $\mathcal{P}^*$ for a vertex set V. Then, every mobile spanner N can host a path system $\mathcal{P}'$ such that the induced load $\ell(e)$ in N is bounded by $\ell(e) \leq c' \ C^* \ \log n$ for a positive constant c'.*

Proof: For the routing in the mobile spanner we reroute all messages of an edge $e = (u, w)$ of the path system $\mathcal{P}^*$ to the shortest path $p(u, w)$ in the mobile spanner N between u and w. A direct implication of the spanner property is that for all nodes q of $p(u, w)$ we observe $|q, u|_\alpha \leq c \cdot |u, w|_\alpha$ and $|q, w|_\alpha \leq c \cdot |u, w|_\alpha$ for $\alpha \in \{2, \mathsf{am}\}$ and constant $c > 1$.

Now we increase the transmission radius of all nodes in the optimal path system by a constant factor c. Then all nodes that will be used for re-routing suffer interferences with the original paths. And the congestion of an edge of the detour can be used as an upper bound for the communication load induced by all detours.

We have seen in Lemma 7 that the congestion increases only by a logarithmic factor if the the transmission radius is increased by a constant factor (if the maximum relative distance and relative speed is polynomial in K_α). ■

In the last lemma we have shown that the number of packets transfered on detours is at most $O(\log n)$ higher than the congestion in the optimal network. We denote by $\text{Int}_\alpha(G)$ the maximum number of interferences in the mobile spanner G. If the loads of all interfering edges can be bounded by m, then the overall congestion is at most $m \cdot \text{Int}_\alpha(G)$, which proves Theorem 2.