Contention resolution protocols

Several processes have access to one resource. Only one request can be satisfied at each step.

Multiaccess channels: Several stations share a broadcast medium; only one message can be broadcast at a given time. E.g., radio wavelength, Ethernet, Aloha, satellite channels, ..

Acknowlege based protocols

- Only one message can be sent at a given time.
- If more than one message is sent, no message is received.
- A station can check if the "channel" is free.
- A station can detect if the message it sent was broadcast successfully.
- A station's protocol uses only that station's history of success and failure transitions - no knowledge about other stations, or even the number of other active stations.

Backoff Protocols

Each station follows the following protocol.

1. The backoff counter b is initially set to 0.

2. **while** the station queue is not empty **do**

   (a) with probability $1/f(b)$ try to broadcast.
   (b) **If** the broadcast succeeded $b = 0$, else $b = b + 1$.

Exponential backoff: $f(b) = 2^b$ (The Ethernet uses $2^{min[b,10]}$).

Linear backoff: $f(b) = \alpha b + 1$.

polynomial backoff: $f(b) = b^\alpha$.

Mathematical model

Under what condition the backoff protocols have good performance?

Time is partitioned into steps.

At a beginning of a step each station receives a new message with probability λ_i.

N stations, $\lambda = \sum_{i=1}^{N} \lambda_i$.

Messages are kept in infinite queues.

System starts with empty queues.

Acknowledgment of a collision or successful transmission is taking place within one step.

Stability

$W_{avg}(t)$ = average waiting time of packets that entered the system at time t.

$L(t)$ = the number of packets in queues at the end of step t.

A system is stable if both $E[W_{avg}(t)]$ and $E[L(t)]$ are bounded with respect to t.

Little's Lemma:

$$E[L(t)] = \lambda E[W_{avg}(t)]$$

Relevance of the Model to Reality

- Upper bound on backoff counter.
- Termination of undelivered messages.
- Arrival distribution.
- Selection of waiting time before attempted retransmission.
- Message length.
- Synchronization.

Stability of the Exponential Backoff Protocol

Theorem 1. *The exponential backoff protocol is unstable for any* $N > 2$ *when the arrival rate at each station is* λ/N, *for* $\lambda \geq \lambda_0 \approx 0.6$.

Potential Function Argument

Define a potential function of the system at time t:

$$\Phi(t) = C \sum_{i=1}^{N} q_i + \sum_{i=1}^{N} 2^{b_i} - N$$

where $C = 2N - 1$.

$q_i =$ number of packets queued at station i at time t.

$b_i =$ the value of the backoff counter at station i at time t.

Plan: Show that $E[\Phi(t)]$ increases by a constant δ in each step.

Lemma 1. *If there is a constant $\delta > 0$ (independent of t) such that for all $t > 0$*

$$E[\Delta\Phi(t)] = E[\Phi(t) - \Phi(t-1)] \geq \delta$$

then the system is unstable.

Proof. A packet enters the queue of station i at time t with probability $\lambda_i = \lambda/N$.

The expected wait time of this packet is at least $q_i + 2^{b_i}$.

For vectors $\bar{q}$ and $\bar{b}$, let $P(t, \bar{q}, \bar{b})$ be the probability that at time t, and for some station $i = 1, \ldots, N$, the queue of station i has q_i items and the counter of i is b_i.

$$E[W_{avg}(t)] \geq \sum_{q,b} \sum_{i=1}^{N} \lambda_i (q_i + 2^{b_i}) P(t, \bar{q}, \bar{b})$$

$$E[\Phi(t)] = \sum_{q,b} \sum_{i=1}^{N} (C q_i + 2^{b_i} - 1) P(t, \bar{q}, \bar{b})$$

Thus,

$$E[W_{avg}(t)] \geq \frac{\lambda}{CN} \Phi(t) \geq t\delta \frac{\lambda}{CN} \Phi(0)$$

$$E[W_{avg}(t)] \to \infty \text{ as } t \to \infty$$

□

Let

$$E[\Delta\Phi(t)] = \sum_{i=1}^{N} Q_i^+ - \sum_{i=1}^{N} Q_i^- + \sum_{i=1}^{N} B_i^+ \sum_{i=1}^{N} B_i^-$$

$Q_i^+(Q_i^-)$ = expected increase (decrease) in potential due to arrivals (departures) to (from) i

$B_i^+(B_i^-)$ = expected increase (decrease) in potential due to counter at i.

At a given step t:

Let $0 \leq M \leq N$ denote the number of stations with non-zero backoff counter.

W.l.o.g $b_1, \ldots, b_M \neq 0$ and $b_{M+1}, \ldots, b_N = 0$.

If $b_i \neq 0$ then $q_i \neq 0$.

All but one of the stations $M+1, \ldots, N$ must have $q_i = 0$.

W.l.o.g we assume that if there is a station with $b_i = 0$ and $q_i \neq 0$ then it's station $M+1$.