Diameter

Theorem 1. *For any t, such that $t/N \to \infty$, w.h.p. the largest connected component of G_t has diameter $O(\log N)$.*

In particular, if the network is connected (which has probability $1 - O(\frac{\log^2 N}{N})$) then w.h.p. its diameter is $O(\log N)$.

Proof Ideas:

- reconnect connections: long-range, random.
- good cache nodes: many reconnect connections.
- many good cache nodes.
- distance between any two c-nodes is $O(\log N)$.

Good Cache Nodes

A cache node is good if during its time in cache it receives at least f (= fixed constant) connections such that:

- they are reconnect connections;
- they are not preferred connections;
- they resulted from different nodes leaving the network.

Color the above connections blue; Color all other connections red.

Lemma 1. *Let node* v *enter the cache at time* t*, where* $t/N \to \infty$*. Then for a sufficiently large choice of the constant* C*,* $\Pr($ *v leaves the cache as a good node*$) \geq 1/2$*.*

The f *blue edges are distributed uniformly at random among the nodes in the current network.*

Furthermore, the probability that a c-node is good is independent of other c-nodes.

Proof

Consider the interval of time in which v was a cache node.

New nodes join the network according to a Poisson process with rate 1. The expected number of connections to v from a new node is D/K.

Nodes leave the network according to a Poisson process with rate $1 - o(1)$. The expected number of connections to v as a result of a old node leaving the network is

$$\approx \sum_{u \in V} \frac{d(u)}{|V|} \frac{D}{d(u)} \frac{1}{K} \approx \frac{D}{K} < 1$$

Each connection to v has a constant probability of being a reconnect connection.

The expected number of connections from an old node u to v in unit time is $\approx \frac{d(u)}{N} \frac{D}{d(u)} = D/N$.

For a sufficiently large C, the lemma holds.

Expansion Lemma

For a c-node v :

- $T_0(v)$: arbitrary connected cluster of $O(\log N)$ c-nodes including v , using red edges.

- $T_i(v)$: c-nodes that are connected by blue edges to $T_{i-1}(v)$, but are not in $T_0(v), \ldots, T_{i-1}(v)$. .

Lemma 2. *If* $|\Gamma_{i-1}(v)| = o(N)$,

$$Pr\{|\Gamma_i(v)| \geq 2|\Gamma_{i-1}(v)|\} \geq 1 - 1/N^5.$$

Proof

Let $W = \Gamma_{i-1}(v)$, $w = |W|$, and let $z \notin W \cup (\cup_{j=0}^{i-1} \Gamma_j(v))$.

The probability that z is connected to W is at least $\frac{1}{2}\frac{fw}{N}(1 - o(1))$.

$E[\Gamma_i(v)] = \frac{f}{2} w(1 - o(1))$.

We use a martingale technique to prove that $\Gamma_i(v)$ is concentrated around its mean with high probability.