Why Randomized Routing

Oblivious routing: The path of one packet does not depend on the source and destination of any other packet in the system.

Lower bound for deterministic oblivious routing.

Theorem 1. *For any N-node network with maximum degree d, and any deterministic oblivious packet routing algorithm, there is a permutation that requires $\Omega(\sqrt{N/d^3})$ steps.*

Proof

Given a network and a deterministic oblivious algorithm we construct a permutation π on the N addresses, such that in routing π there is some vertex v that is traversed by at least $\sqrt{N/d}$ packets. Since the degree is bounded by d, it implies that routing takes $\Omega(\sqrt{N/d^3})$.

A deterministic oblivious algorithm is defined by N^2 routes, where $P_{u,w}$ gives the route taken by a packet from u to w.

To construct a "bad" permutation we need a vertex x such that for many destinations $v_1, \ldots, v_k$ many origins $w_1, \ldots, w_k$ send their packets through x.

Let S_k^v be the set of vertices such that at least k paths to v enter each of these vertices.

Lemma 1.

$$|S_k^v| \geq \frac{N}{d(k-1)+1}$$

Proof. Clearly $v \in S_k^v$ since $N-1$ paths to v enter v.

All paths from an origin outside S_k^v must enter S_k^v.

The set S_k^v has no more than $d|S_k^v|$ direct neighbors outside the set.

By the definition of S_k^v no more than $k-1$ paths to v can enter any vertex that is not in the set.

Thus, at most $|S_k^v|d(k-1)$ paths can enter the set S_k^v. The remaining paths must start inside the set.

Thus,

$$|S_k^v| + |S_k^v|d(k-1) \geq N$$

□

We need a vertex x that is included in many S_k^v for many v's.

$$\sum_{v \in V} |S_k^v| \geq \frac{N^2}{d(k-1)+1}$$

Since there are only n vertices, each vertex is "on average" in $\frac{N}{d(k-1)+1}$ sets.

Thus there is at least one vertex x, and $l = \frac{N}{d(k-1)+1}$ vertices $v_1, \ldots, v_l$ such that $x \in S_k^{v_i}$ for $i = 1, \ldots, l$.

For each of the l vertices there are k origins that uses x on the path to v_i.

Fix $k = \sqrt{N/d}$. then $l \geq \sqrt{N/d}$.

We can construct a permutation with k paths that traverse vertex x.

Store and Forward switching

- Messages are conveyed as *packets*.
- An entire packet can reside at a routing switch.
- Packets are sent from a queue of one switch to queue of another switch until they reach their destinations.
- At most one packet is transmitted on a link in each step.
- A link can be shared by different sessions using that link, but the sharing is on a demand basis, rather than a fixed allocation basis.

Circuit Switching

When a session (or connection) is initiated, bandwidth is allocated along a path connecting the two endpoints for the duration of the connection.

Virtual circuit routing is store-and-forward switching in which a particular path is set up when a session is initiated and maintained during the life of the session. Like circuit switching, but links are shared by sessions on a demand basis.

Dynamic routing or *datagram routing* is store and forward switching in which each packet finds it own path through the network using the current information available at the nodes visited.