

Problem of the week #6:

Given - Feb. 20, 2003; Due - Feb. 27, 2003 (in class).

Use the Lovasz Local Lemma to show that every k -regular directed graph has a collection of $\lfloor \frac{k}{3 \ln k} \rfloor$ vertex-disjoint directed cycles.

A k -regular directed graph has an out-degree (and in-degree) of k for every vertex.

(Hint: Randomly partition V into $\lfloor \frac{k}{3 \ln k} \rfloor$ parts and show that with positive probability each part contains a directed cycle.)

You might need the fact that if every vertex in a graph has an out-degree of at least 1, then there is a directed cycle in the graph.)