

Problem of the week #9:

Given - Apr. 1, 2003; Due - Apr. 8, 2003 (in class).

A coloring of a graph G with vertex set $V = \{1, \dots, n\}$ is an assignment of colors to the vertices of the graph such that no two adjacent vertices receive the same color; the *chromatic number* of G , denoted $\chi(G)$, is the minimum number of colors needed for this purpose.

Consider a random graph generated by the $G_{n,p}$ model; i.e., there are n vertices and each edge is independently chosen with probability p . Show that

$$\Pr(|\chi(G) - E[\chi(G)]| > \lambda\sqrt{n}) \leq 2e^{-\lambda^2/2}$$

Hint: Model the chromatic number as a function of n arguments and show that it satisfies the Lipschitz condition.