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## Task

Implement the diagonal matrix type which contains integers. These are square matrices that can contain nonzero entries only in their diagonal. Don't store the zero entries. Store only the entries that can be nonzero in a sequence. Implement as methods: getting and setting the entry located at index ( $i, j$ ), adding and multiplying two matrices, reading and printing the matrix (in a square shape).

## Diagonal matrix type

## Set of values

$\operatorname{Diag}(n)=\left\{a \in \mathbb{Z}^{\mathrm{n} \times \mathrm{n}} \mid \forall i, j \in[1 . . n]: i \neq j \rightarrow a[i, j]=0\right\}$

## Operations

## 1. Getting an entry

Getting the entry of the $i$ th column and $j$ th row $(i, j \in[1 . . n]): e:=a[i, j]$.
Formally:

$$
\begin{aligned}
& A: \operatorname{Diag}(n) \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} \\
& \quad a \quad i \quad j \quad e \\
& \text { Pre }=\left(a=a, \wedge i=i^{\prime} \wedge j=j^{\prime} \wedge i, j \in[1 . . n]\right) \\
& \text { Post }=(\text { Pre } \wedge e=a[i, j])
\end{aligned}
$$

This operation needs any action only if $i=j$, otherwise the output is zero.

## 2. Setting an entry

Setting the entry of the $i$ th column and $j$ th row $(i, j \in[1 . . n]): a[i, j]:=e$. Entries outside the diagonal cannot be modified ( $i=j$ ).

Formally: $\quad A=\operatorname{Diag}(n) \times \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$
$a \quad i \quad j \quad e$
Pre $=\left(e=e^{\prime} \wedge a=a^{\prime} \wedge i=i^{\prime} \wedge j=j^{\prime} \wedge i, j \in[1 . . n] \wedge i=j\right)$ Post $=\left(e=e^{\prime} \wedge i=i^{\prime} \wedge j=j^{\prime} \wedge a[i, j]=e \wedge \forall k, l \in[1 . . n]:(k \neq i \vee l \neq j) \rightarrow a[k, l]=a \prime[k, l]\right)$

This operation needs any action only if $i=j$, otherwise it gives an error if we want to modify a zero entry.
3. Sum

Sum of two matrices: $c:=a+b$. The matrices have the same size.

$$
\begin{aligned}
\text { Formally: } & A=\operatorname{Diag}(n) \times \operatorname{Diag}(n) \times \operatorname{Diag}(n) \\
& a \quad b \quad c \\
& \text { Pre }=\left(a=a^{\prime} \wedge b=b^{\prime}\right) \\
& P o s t=(\operatorname{Pre} \wedge \forall i, j \in[1 . . n]: c[i, j]=a[i, j]+b[i, j])
\end{aligned}
$$

In case of diagonal matrices there is an easier version:

$$
\forall i \in[1 . . n]: c[i, i]=a[i, i]+b[i, i] \text { és } \forall i, j \in[1 . . n]: i \neq j \rightarrow c[i, j]=0 .
$$

## 4. Multiplication

Multiplication of two matrices: $c:=a * b$. The matrices have the same size.
Formally: $\quad A=\operatorname{Diag}(n) \times \operatorname{Diag}(n) \times \operatorname{Diag}(n)$

$$
\begin{aligned}
& a \quad b \quad c \\
& \text { Pre }=\left(a=a^{\prime} \wedge b=b^{\prime}\right) \\
& \text { Post }=\left(\text { Pre } \wedge \forall i, j \in[1 . . n]: c[i, j]=\sum_{k=1 . . n} a[i, k] * b[k, j]\right)
\end{aligned}
$$

In case of diagonal matrices there is an easier version:

$$
\forall i \in[1 . . n]: c[i, i]=a[i, i] * b[i, i] \text { és } \forall i, j \in[1 . . n]: i \neq j \rightarrow c[i, j]=0 .
$$

## Representation

Only the diagonal of the $n \times n$ matrix has to be strored.

$$
a=\begin{array}{ccccc}
a_{11} & 0 & 0 & \ldots & 0 \\
0 & a_{22} & 0 & \ldots & 0 \\
0 & 0 & a_{33} & \ldots & 0 \\
0 & 0 & 0 & \ldots & a_{n n}
\end{array} \quad \leftrightarrow \quad v=\left\langle\begin{array}{llll}
a_{11} & a_{22} & a_{33} & a_{n n}
\end{array}\right\rangle
$$

Only a one-dimension array ( $v$ ) is needed, with the help of which any entry of the diagonal matrix can be get:

$$
a[i, j]=\left\{\begin{array}{ccc}
v[i] & \text { if } & i=j \\
0 & \text { if } & i \neq j
\end{array}\right\}
$$

## Implementation ${ }^{1}$

## 1. Getting an entry

Getting the entry of the $i$ th column and $j$ th row $(i, j \in[1 . . n]) \quad e:=a[i, j]$ where the matrix is represented by $v, 1 \leq i \leq n$, and $n$ stands for the size of the matrix can be implemented as

| $i=j$ |  |
| :---: | :---: |
| $e:=v[i-1]$ | $e:=0$ |

## 2. Setting an entry

Setting the entry of the $i$ th column and $j$ th row ( $i, j \in[1 . . n]$ ) $a[i, j]:=e$ where the matrix is represented by $v, 1 \leq i \leq n$, and $n$ stands for the size of the matrix can be implemented as

| $i=j$ |  |
| :--- | :--- |
| $\nu[i-1]:=e$ | $S K I P$ |

## 3. Sum

The sum of matrices $a$ and $b$ (represented by arrays $t$ and $u$ ) goes to matrix $c$ (represented by array $u$ ), where all of the arrays have to have the same size.

$$
\forall i \in[0 . . n-l]: u[i]:=v[i]+t[i]
$$

## 4. Multiplication

The product of matrices $a$ and $b$ (represented by arrays $t$ and $u$ ) goes to matrix $c$ (represented by array $u$ ), where all of the arrays have to have the same size.

$$
\forall i \in[0 . . n-l]: u[i]:=v[i] * t[i]
$$

[^0]
## Testing

## Testing the operations (black box testing)

1) Creating, reading, and writing matrices of different size.
a) $0,1,2,5$-size matrix
2) Getting and setting an entry
a) Getting and setting an entry in the diagonal
b) Getting and setting an entry outside the diagonal
c) Illegal index, indexing a 0 -size matrix
3) Copy constructor
a) Creating matrix $b$ based on matrix $a$, comparing the entries of the two matrices. Then, changing one of the matrices and comparing the entries of the two matrices.
4) Assignment operator
a) Executing command $b=a$ for matrices $a$ and $b$ (with and without same size), comparing the entries of the two matrices. Then, changing one of the matrices and comparing the entries of the two matrices.
b) Executing command $c=b=a$ for matrices $a, b$, and $c$ (with and without same size), comparing the entries of the three matrices. Then, changing one of the matrices and comparing the entries of the three matrices.
c) Executing command $a=a$ for matrix $a$.
5) Sum of two matrices, command $c:=a+b$.
a) With matrices of different size (size of $a$ and $b$ differs, size of $c$ and $a$ differs)
b) Checking the commutativity $(a+b==b+a)$
c) Checking the associativity $(a+b+c==(a+b)+c=a+(b+c))$
d) Checking the neutral element ( $a+0==a$, where 0 is the null matrix)
6) Multiplication of two matrices, command $c:=a^{*} b$.
a) With matrices of different size (size of $a$ and $b$ differs, size of $c$ and $a$ differs)
b) Checking the commutativity ( $\mathrm{a} * \mathrm{~b}==\mathrm{b} * \mathrm{a}$ )
c) Checking the associativity ( $\mathrm{a} * \mathrm{~b} * \mathrm{c}==(\mathrm{a} * \mathrm{~b}) * \mathrm{c}==\mathrm{a} *(\mathrm{~b} * \mathrm{c}))$
d) Checking the neutral element ( $\mathrm{a} * 0==0$, where 0 is the null matrix)
e) Checking the identity element ( $\mathrm{a} * 1==\mathrm{a}$, where 1 is the identity matrix)

Testing based on the code (white box testing)

1. Creating an extreme-size matrix $(-1,0,1,1000)$.
2. Generating and catching exceptions.

[^0]:    ${ }^{1}$ To implement an operation, a program has to be given (not necessarily structogram).

